Worksheet 6

Math 392, Abstract Algebra

Friday, January 15, 2021

Definition from lecture: an ideal $P \subsetneq R$ is called *prime* if $rs \in P$ implies $r \in P$ or $s \in P$.

Let $R = \mathbb{Z}[x]$ and let $P = \langle 2, x \rangle$. On worksheet 3 you saw that P is the set of polynomials whose constant term is even.

1. Let $\phi\colon R \to \mathbb{Z}_2$ be the map given by

$$\phi(a_n x^n + \cdots + a_1 x + a_0) = \bar{a}_0.$$

   Convince yourselves that ϕ is a homomorphism.

2. Convince yourselves that $\ker \phi = P$.

   (That is, for all $f \in R$ we have $\phi(f) = \bar{0}$ if and only if $f \in P$.)

3. Use the previous problem to prove that P is prime.

4. Challenge: Prove that the ideal $I = \langle 2, x^2 + 5 \rangle \subset R$ is not prime, by showing that $(x+1)(x-1) \in I$ but $x + 1 \notin I$ and $x - 1 \notin I$.