

Midterm

Math 420

April 29, 2016

Use your own notebook paper. Start each problem on a new sheet of paper.

1. Let $A = \begin{pmatrix} 5 & 1 \\ 2 & 4 \end{pmatrix}$.
 - (a) Find the eigenvalues of A .
 - (b) Find eigenvectors for those eigenvalues.
 - (c) Find the general solution of $X' = AX$.
 - (d) Sketch the phase portrait.
 - (e) If $X(t)$ is a solution of $X' = AX$ with $X(0) = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$, find $X(1)$. What quadrant is it in? Does this agree with your picture?
2. Consider the second-order equation $x'' + x' - 2x = 0$.
 - (a) Convert this to a first-order system in two variables, find the eigenvalues and eigenvectors, and sketch the phase portrait.
 - (b) For which initial conditions, i.e. for which values of $x(0)$ and $x'(0)$, do we have $x(t) \rightarrow \infty$ as $t \rightarrow \infty$? For which do we have $x(t) \rightarrow 0$? For which do we have $x(t) \rightarrow -\infty$?
3. Let $A = \begin{pmatrix} \lambda & 1 \\ 0 & \lambda + \epsilon \end{pmatrix}$.
 - (a) Find the eigenvalues of A .
 - (b) If $\epsilon > 0$, find eigenvectors for those eigenvalues.
 - (c) Let T be the matrix whose columns are those eigenvectors. Find T^{-1} . Check that $T^{-1}T = I$ or $TT^{-1} = I$.
 - (d) Check that $T^{-1}AT = \begin{pmatrix} \lambda & 0 \\ 0 & \lambda + \epsilon \end{pmatrix}$.
 - (e) Show that if $\epsilon = 0$ then there is no T such that $T^{-1}AT = \begin{pmatrix} \lambda & 0 \\ 0 & \lambda \end{pmatrix}$.
Hint: What is $T \begin{pmatrix} \lambda & 0 \\ 0 & \lambda \end{pmatrix} T^{-1}$?
 - (f) What happens to T^{-1} from part (d) as $\epsilon \rightarrow 0$?