Midterm 2

Friday, November 9, 2018

Use your own notebook paper, or get some from me. Put each problem on its own page. Each part is worth 5 points, for a total of 50 points.

1. (a) Let X be a set. Define what it means for a collection T of subsets of X to be a topology.

   (b) Let $X = \mathbb{R}$, and let T consist of $\varnothing$, all of $\mathbb{R}$, and sets of the form $[a, \infty)$ for $a \in \mathbb{R}$. Show that T is not a topology.

2. (a) Let X and Y be topological spaces. Define what it means for a map $f \colon X \to Y$ to be continuous.

   (b) Define a map $f \colon \mathbb{R} \to \mathbb{R}$ by

   $$f(x) = \begin{cases} 1/x & \text{if } x \neq 0, \\ 0 & \text{if } x = 0. \end{cases}$$

   Show, using your definition from part (a), that f is not continuous if we put the usual topology on both copies of $\mathbb{R}$.

3. (a) Let X be a topological space and $A \subset X$. Define the closure $\bar{A}$ and the interior $\operatorname{int}(A)$.

   (b) Let $X = \mathbb{R}$ with the lower-limit topology: that is, the open sets are $\varnothing$, all of $\mathbb{R}$, and sets of the form (a, ∞) for $a \in \mathbb{R}$, or equivalently, the closed sets are $\varnothing$, all of $\mathbb{R}$, and sets of the form $(-\infty, a]$ for $a \in \mathbb{R}$.

   Let $A = (0, 1)$. Find $\bar{A}$ and $\operatorname{int}(A)$ in the lower-limit topology.

(continued on next page)

4. (a) Let X be a topological space and $Y \subset X$. Define what it means for a subset $V \subset Y$ to be open in the subspace topology. State what we proved about when a subset $G \subset Y$ is closed in the subspace topology. (Do not give the proof.)

   (b) Let $A \subset Y$, let $\mathrm{cl}_X(A)$ denote the closure of A in X, and let $\mathrm{cl}_Y(A)$ denote the closure of A in Y. Show that

$$\mathrm{cl}_Y(A) = \mathrm{cl}_X(A) \cap Y.$$

   (c) Let $A \subset Y$, and let $\mathrm{int}_X(A)$ denote the interior of A as a subset of X, and $\mathrm{int}_Y(A)$ denote the interior of A as a subset of Y. Show that

$$\mathrm{int}_X(A) \subset \mathrm{int}_Y(A).$$

   (d) Give an example where the inclusion in part (c) is strict.