1. Let $X$ be a topological space. Define a relation $\sim$ on $X$ by declaring that $p \sim q$ iff there is a path from $p$ to $q$, that is, a continuous map $\gamma : [0,1] \rightarrow X$ such that $\gamma(0) = p$ and $\gamma(1) = q$.

(a) Show that this is an equivalence relation: reflexive, symmetric, and transitive. Hint: The interesting one is transitive.

(b) The equivalence classes of this equivalence relation are called are called path components. Describe (without proof) the path components of the following spaces:
   i. $\{(x, y) \in \mathbb{R}^2 : xy > 1\}$.
   ii. $\mathbb{Q}$.
   iii. The topologist’s sine curve.

2. (a) Let $X$ be a compact space, and let $F_1 \supset F_2 \supset F_3 \supset \cdots$ be a descending chain of non-empty closed subsets. Show that the intersection $F_1 \cap F_2 \cap F_3 \cap \cdots$ is not empty.
   Hint: Otherwise $X \setminus F_1, X \setminus F_2, \ldots$ is an open cover of $X$.

(b) Give an example of a non-compact space $X$ and a descending chain of closed subsets $F_1 \supset F_2 \supset F_3 \supset \cdots$ whose intersection is empty.

3. What is one question you have about last week’s lectures?