

Homework 8

Due Wednesday, March 8, 2017

1. Let D_6 be the dihedral group of the hexagon, which has $12 = 2^2 \cdot 3$ elements.
 - (a) List all Sylow 2-subgroups of D_6 , i.e. subgroups of order $2^2 = 4$. Confirm that they are all conjugate to one another, and that the number n_2 of such subgroups satisfies $n_2 \equiv 1 \pmod{2}$ and $n_2 \mid 3$.
 - (b) List all Sylow 3-subgroups of D_6 . Confirm that they are all conjugate to one another, and that $n_3 \equiv 1 \pmod{3}$ and $n_3 \mid 4$.
2. Let G be the group of order 12 from Homework 4, problem 6 and Homework 7, problem 2:

$$G = \{ 1, a, a^2, a^3, a^4, a^5, \\ b, ba, ba^2, ba^3, ba^4, ba^5 \},$$

subject to the relations

$$a^6 = 1 \qquad b^2 = a^3 \qquad ab = ba^{-1}.$$

- (a) List all Sylow 2-subgroups of G . Confirm that they are all conjugate to one another, and that $n_2 \equiv 1 \pmod{2}$ and $n_2 \mid 3$.
 - (b) List all Sylow 3-subgroups of G . Confirm that they are all conjugate to one another, and that $n_3 \equiv 1 \pmod{3}$ and $n_3 \mid 4$.
3. Optional: List all Sylow 2- and 3-subgroups of S_4 . Exhibit two subgroups of order 4 (not 8!) that are not isomorphic, hence not conjugate.
 4.
 - (a) Let G be a group and let $K \subset H \subset G$ be subgroups. Show that if K is normal in G then K is normal in H .
 - (b) Let $G = D_4$, let $H = \{1, s, r^2, sr^2\}$, and let $K = \{1, s\}$. Show that H is normal in G , and K is normal in H , but K is not normal in G . Hint: Index 2.