Earlier: reduction of the structure group to $G \subset GL_n\mathbb{R}$

Let X, Y be two Riem. manifolds

a map $F: X \to Y$ is <u>conformal</u> if $F^* g_Y = \text{function} \cdot g_X$

F preserves angles but not lengths

a conformal structure $\longleftrightarrow$ "conformal group" $\subset GL_n\mathbb{R}$

$$CO(n) = O(n) \times \mathbb{R}_+ = \{ A \in GL_n\mathbb{R} \mid A^T A = \text{const.}\, 1 \}$$

Lie alg: $A^T + A = \text{const.}$ $\begin{pmatrix} a & & + \\ & a & \\ - & & a \end{pmatrix}$

$A = \text{skew} + \text{scalar}$

arithmetic people sometimes call this $GO(n)$?

Fun: $CO(2) \cap GL_2^+ = GL_1\mathbb{C} \subset GL_2\mathbb{R}$

<u>Question:</u> what does integrable mean for this group?

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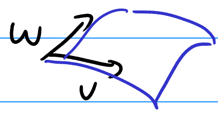
covariant derivatives $\longleftrightarrow$ parallel transport $\longleftrightarrow$ splittings $TE \cong \text{horz} \oplus \text{vert}$.

curvature: measures failure of ∇_v to commute with ∇_w = PT around a square, lim as $\square \to \cdot$ = measures failure of $[\tilde{v}, \tilde{w}]$ to be horz, where $\tilde{v}, \tilde{w}$ are horz. lifts of v, w.

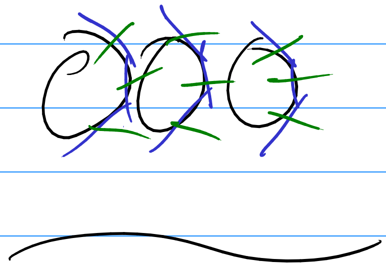


Read Poor 2.66 page 82.

Last time: principal G-bundle $\quad G \hookrightarrow P \xrightarrow{\pi} X$

$$0 \to \text{vertical bundle} \to TP \xrightarrow{D\pi} \pi^* TX \to 0$$

connection is a G-inv splitting

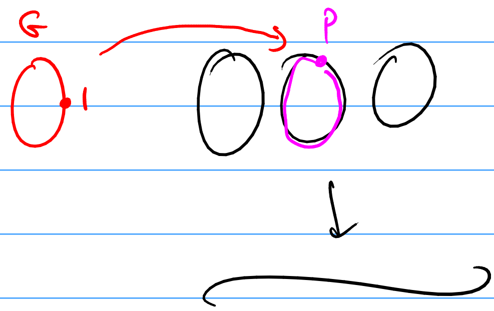


allows you to write $TP = \text{vert} \oplus \pi^* TX$
and to regard $\pi^* TX$ as a sub-bundle of TP, not just a quot.

vertical bundle $\cong \mathcal{O}_P \otimes \mathfrak{g}$ $\qquad \mathfrak{g} =$ Lie alg of G

at $p \in P$, get a map

$$G \longrightarrow P$$
$$g \longmapsto p \cdot g$$



and its derivative

$$\mathfrak{g} := T_1 G \hookrightarrow T_p P \qquad \text{image} = \text{vert}|_p$$

but vertical bundle is not trivial as a G-equivar vector bundle

instead, if $R_g : P \to P$ is the map
$$p \longmapsto p \cdot g$$

then R_{g*} acts on fiber $\mathfrak{g}$ of the vert. bundle

by adjoint action. $\qquad \mathfrak{g} \longrightarrow \mathfrak{g}$
$$A \longmapsto gAg^{-1} \text{ or maybe } g^{-1}Ag.$$

Given a connection on P,

$TP \cong \text{vert} \oplus \text{horz}$
$\cong \underline{\mathfrak{g}} \oplus \pi^* TX$

have the connection form
$\omega \in \Gamma(T^*P \otimes \mathfrak{g})$

$\omega(\text{vector field on } P) = \text{vertical part}$

then horizontal sub-bundle $\subset TP$
is the kernel of ω.

for $V, W \in \Gamma(TX)$, get $\overset{\text{curvature}}{\Omega(V,W)} = -\omega([\tilde{V}, \tilde{W}])$
where $\tilde{v}, \tilde{w} \in \Gamma(TP)$ are horizontal lifts.

more gen, for two v.f.'s $V, W \in \Gamma(TP)$,

define curvature form $\Omega(V,W) = -\omega([\text{horz}(V), \text{horz}(W)])$

equation: $\Omega(V,W) = d\omega(V,W) + [\omega(V), \omega(W)]$

maybe next time?
idea: 4 cases: $V = \text{horz}$ or vert
$W = \text{horz}$ or vert

$$d\omega(V,W) = V(\omega(W)) - W(\omega(V)) - \omega([V,W])$$