Last time, X = mfld of dim n

E = $\mathbb{C}x$ v.b. of rank r

s = transverse section of E

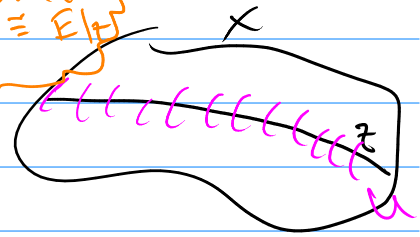
cuts out $Z \subset X$ of dim $n-2r$

Claim: for $\eta \in H^{n-2r}_{dR}(X)$

have $\int_X \eta \wedge c_r(E) = \int_Z \eta|_Z$

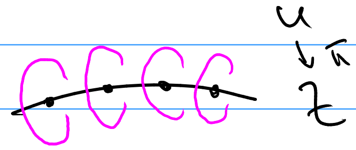
$\mathcal{U} \cong$ open disc bundle in $N_{Z/X} \cong E|_Z$

Chose a tubular nbd $\mathcal{U}$ of Z and a connection ∇ on E s.t. $c_r(\nabla)$ is supported in $\mathcal{U}$.



let $\pi : \mathcal{U} \to Z$. Can replace η with $\pi^*\alpha$ where $\alpha = \eta|_Z$

(claim: $\eta - \pi^*\alpha$ is exact. Poincaré lemma essentially)



if you want to read, check out Bott + Tu

Today: Consider $X' = \mathbb{P}(\mathcal{O}_Z \oplus E|_Z)$

$\pi : X' \to Z$ a $\mathbb{CP}^r$-bundle

on X', have $E' := \mathcal{O}_\pi(1) \otimes \pi^* E|_Z$

it has a standard section s' that cuts out a copy of $Z \subset X'$ transversely..

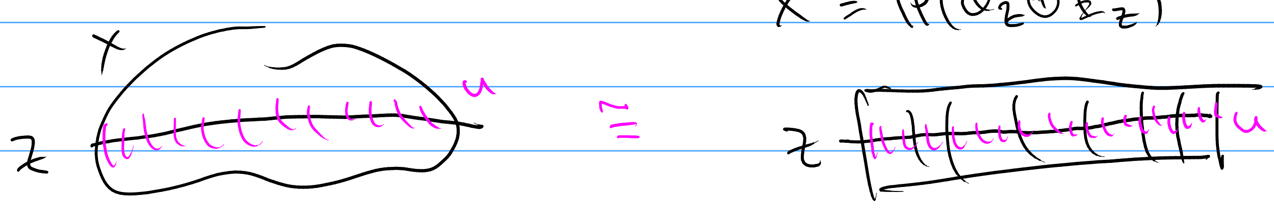
fiberwise, get $(1:0:\cdots:0) \in \mathbb{CP}^r$

for $\alpha \in H^{n-2r}_{dR}(Z)$, $\quad \int_{X'} c_r(E') \wedge \pi^*\alpha$

$$= \int_Z \pi_* c_r(E') \wedge \alpha$$

and $\int_{\mathbb{CP}^r} c_{top}(E'|_{Fiber}) = 1$ so $\pi_* c_r(E') = 1$

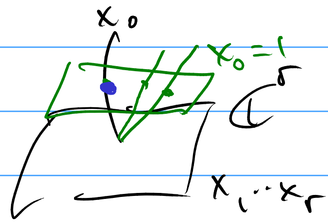
$X' = \mathbb{P}(\mathcal{O}_Z \oplus E_Z)$



$\mathbb{C}^r \hookrightarrow \mathbb{CP}^r = \mathbb{C}^{r+1} \setminus 0 / \mathbb{C}^*$

$(x_1, \dots, x_r) \longmapsto (1 : x_1, \dots, x_r)$

$(0, \dots, 0) \longmapsto (1 : 0 \cdots 0)$



cut out by a transverse section of $\mathcal{O}(1)^r$, namely $(x_1, x_2, \dots, x_r)$

[have the inclusion $\mathcal{O}(-1) \hookrightarrow \mathcal{O}^{r+1}$
and the projection $\mathcal{O}^{r+1} \twoheadrightarrow \mathcal{O}^r$
$(x_0, x_1, \dots) \mapsto (x_1, \dots)$

composition $\mathcal{O}(-1) \to \mathcal{O}^r$
vanishes only at $(1, 0, \dots 0)$
and corresp. to a section of $\mathcal{O}(1) \otimes \mathcal{O}^r$)

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replace $\mathbb{C}^r$ with a vector space E_z for $z \in Z$

$E_z \hookrightarrow \mathbb{P}(\underline{\mathbb{C} \oplus} E_z) = \mathbb{C} \oplus E_z \setminus 0 / \mathbb{C}^*$

$v \longmapsto (1 : v)$

the point $(1:\vec{0})$ is cut out by a transverse section of $\mathcal{O}(1)\otimes E_z$

or if you prefer, $\underline{\mathcal{O}(-1)} \hookrightarrow \mathcal{O}_{\mathbb{P}}\otimes(\mathbb{C}\oplus E_z) \twoheadrightarrow \mathcal{O}_{\mathbb{P}}\otimes E_z$

the map $\mathcal{O}(-1)\to\mathcal{O}_{\mathbb{P}}\otimes E_z$ vanishes only at $(1:\vec{0})$ and corresp. to a section of $\mathcal{O}(1)\otimes E_z$.

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in a family: $E|_Z \to Z$

$\rightsquigarrow \underbrace{\mathbb{P}(\mathcal{O}_Z\oplus E|_Z)}_{X'=}$ v.b. whose fiber at $z\in Z$ is $\mathbb{C}\oplus E_z$

$\mathbb{CP}^r$ bundle compactifying that.

$Z\subset X'$ cut out by a section $\mathcal{O}_\pi(1)\otimes\pi^* E|_Z =: E'$

emb. as $(1:\vec{0})$

$c_r(E')\rightsquigarrow$ on a fiber, $c_r(\mathcal{O}(1)^r) = c_1(\mathcal{O}(1))^r$

and we saw last week that $\underline{\underline{\int c_1(\mathcal{O}(1))^r = 1}}$.