Promised last time:
\( t = k\)-manifold

\( T: U \to \mathbb{R} \) open disc bundle
in some vector bundle

\( \eta \) a closed \( k \)-form on \( U \)

w) let \( \alpha = \eta \) (zero section): closed \( k \)-form on \( T \)

then \( \eta = \pi^* \alpha \) is exact.

reason: id: \( U \to U \) and \( U \to T \to U \)
are homotopic via straight-line hg.

Last week:
on \( \mathbb{C} P^r \), the tautological (i.e. \( O(-1) \)
and its dual \( O(1) \). Linear forms on \( \mathbb{C}^{r+1} \)
give sections of \( O(1) \)

let \( h = c_1(O(1)) \)

computed \( \int_{\mathbb{C} P^r} h^r = 1 \), so it deserves to be called \( h \).

observe: \( c_r(O(1)^\otimes r) = h^r \), because

\[
c_r(O(1)^\otimes r) = (c(O(1)))^r = (1 + h)^r
= 1 + rh + \binom{r}{2} h^2 + \cdots + h^r
\]

and this works on the level of forms.
Let $X$ be a $n$-manifold, $E$ a $\mathbb{C}^x$ v.b. of rank $r$, $S$ a transverse section cutting out $T < X$ of codim. $2r$ produced $\pi : X' \to T$ a $\mathbb{CP}^{r-1}$-bundle containing a copy of $T$.

$E'$ a v.b. on $X'$
st. $E'|_{T} = E|_{T}$

and $E'|_{\text{fiber \ } T} = O_{\mathbb{CP}^{r-1}}(1)$

(and a section $s'$ of $E'$ that cut out $T$ transversely...)

Then for $\alpha \in H_{ap}^{*}(T)$, have

$$\int_{X'} c_{r}(E') \wedge \pi^{*}\alpha = \int_{T} \pi^{*}(c_{r}(E)) \wedge \alpha$$

**Real case:**

now $E$ real oriented v.b. of rank $2r$ choose a Riem. metric.

so for any compact $\mathcal{V}$ on $E$, curvature $\mathcal{S}$ acts vol. in skew-symmetric matrices $\mathcal{S} \in \mathbb{M}(\Lambda^{2}T^{*}X \otimes \mathfrak{so}(E))$

put $\mathcal{K}(\mathcal{V}) = \text{PF}(\mathcal{S}), \quad \text{PF}(\mathcal{A}) = (\det \mathcal{A})$
Claim is that for a transverse section \( s \in \mathcal{P}(E) \), and \( \eta \) a closed \((n-2r)\)-form on \( X \),

\[
\int_X \alpha \wedge \eta = \int_{\tau \simeq 0} \eta
\]

Idea: replace \( X \) with a \( S^{2r} \)-bundle

\[ X' \to T \]

How? take the unit disc bundle \( D \subset E \)

collapse the boundary of each disc to get a \( S^{2r} \)-bundle, \( X' \)

(If I collapsed \( 2D \) to a point, that's the Thom space of \( E|_I \).
Think of it as a twisted suspension.)

Want: \( E' \) on \( X' \) s.t. \( E'|_I = E|_I \) and \( E'|_I \) has \( X = I \)

\( \Rightarrow \) too much to ask.

Instead: \( E' = T_{\pi} = T_{X'/I} \) try. bundle to fibers of \( \pi \)
get a section of $E'$ that vanishes (transversely) at north and south pole

then $\chi(TS^{2n}) = 2$ (long computation like we did with $\mathbb{CP}^n$)

and $s'$ cuts out two copies of $\mathbb{C}$ in $\overline{X}'$

$E'/\text{either} \mathbb{C} = E'/\mathbb{C}$

Nail it down more next time...