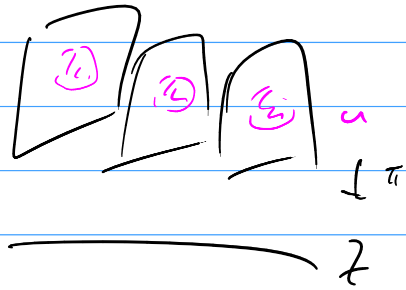


Promised last time:

Z = k-manifold

$\pi: U \to Z$ open disc bundle in some vector bundle

η a closed k-form on U

w/ let $\alpha = \eta|_{\text{zero section}}$: closed k-form on Z

then $\eta - \pi^*\alpha$ is exact.

reason: $id: U \to U$ and $U \xrightarrow{\pi} Z \hookrightarrow U$ are homotopic via straight-line htpy.

last week:

on $\mathbb{CP}^r$, the tautological l.b. $\mathcal{O}(-1)$ and its dual $\mathcal{O}(1)$. linear forms on $\mathbb{C}^{r+1}$ give sections of $\mathcal{O}(1)$

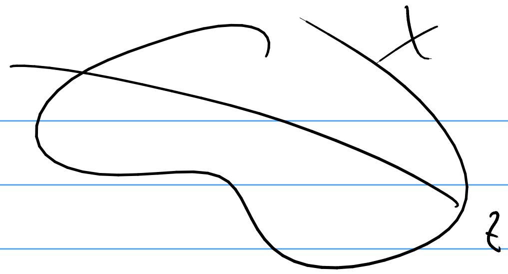
let $h = c_1(\mathcal{O}(1))$

computed $\int_{\mathbb{CP}^r} h^r = 1$, so it deserves to be called h.

observe: $c_r(\mathcal{O}(1)^{\oplus r}) = h^r$, because

$$c(\mathcal{O}(1)^{\oplus r}) = c(\mathcal{O}(1))^r = (1+h)^r = 1 + rh + \binom{r}{2}h^2 + \cdots + h^r$$

and this works on the level of forms.



Last time:

given X = n-manifold

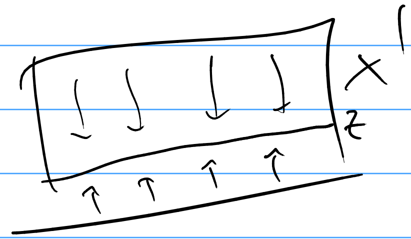
E = a cx v.b. of rank r

s = transverse section

cutting out $Z \subset X$ of codim. $2r$

produced $\pi : X' \longrightarrow Z$ a $\mathbb{CP}^r$-bundle

containing a copy of Z



E' a v.b. on X'

s.t. $E'|_Z = E|_Z$

and $E'|_{\text{fiber of } \pi} = \mathcal{O}_{\mathbb{CP}^r}(1)^r$ ←

(and a section s' of E' that cut out Z transversely...)

then for $\alpha \in H^{top}_{dR}(Z)$, have

$$\int_{X'} c_r(E') \wedge \pi^* \alpha = \int_Z \cancel{\pi_*(c_r(E'))}^{\,1} \wedge \alpha$$

Real case:

now E = real oriented v.b. of rank $2r$

choose a Riem. metric.

so for any compat ∇ on E, curvature Ω

takes vals. in skew-symm. matrices

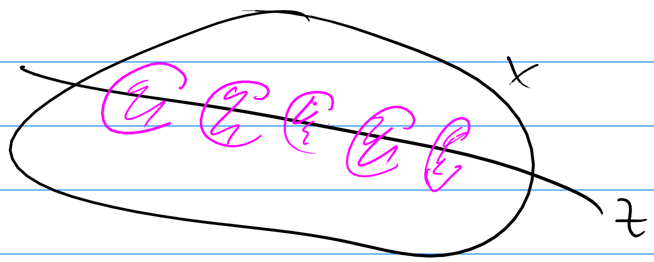
$$\Omega \in \Gamma(\Lambda^2 T^* X \otimes \mathfrak{so}(E))$$

put $\chi(\nabla) = \mathrm{Pf}(\Omega)$ $\qquad$ $\mathrm{Pf}(A)^2 = (\det A)$

Claim is that for a transverse section $s \in \Gamma(E)$,
and η a closed $(n-2r)$-form on X,

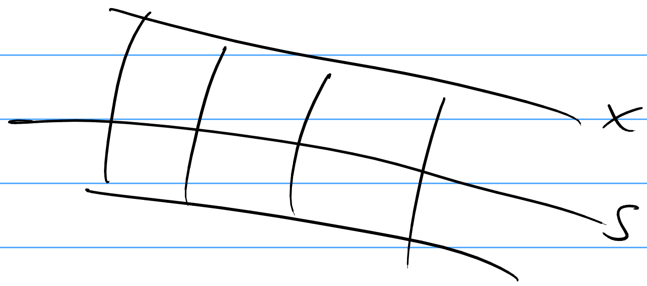
$$\int_X \chi(\nabla) \wedge \eta = \int_{Z=\{s=0\}} \eta$$

idea: replace X



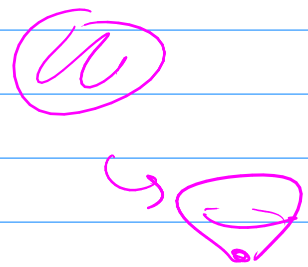
with a S^{2r}-bundle

$X' \longrightarrow Z$



how? take the unit
disc bundle $D \subset E$

collapse the boundary of each disc
to get a S^{2r}-bundle. X'



(if I collapsed ∂D
to a point, that's
the Thom space of $E|_Z$
think of it as a twisted suspension)

want: E' on X' s.t. $E'|_Z = E|_Z$
and $E'|_{\text{fiber } S^{2r}}$ has $\chi = 1$

↳ too much to ask.

instead: $E' = T_\pi = T_{X'/Z}$ tang. bundle to fibers of π

get a section of E' that vanishes (transversely)
at north and south pole

then $\chi(TS^{2r}) = 2$ (long computation like
we did with $\mathbb{CP}^r$)

and s' cuts out <u>two</u> copies of Z in X'

$$E'|_{\text{either } Z} = E|_Z$$

nail it down more next time...