Worksheet 14
Math 607, Homological Algebra
Monday, May 4, 2020

1. We know that \( Z \to \mathbb{Q} \) is flat. Show by example that it is not faithfully flat: that is, find a sequence

\[
0 \to A \to B \to C \to 0
\]

of Abelian groups that is \textit{not} exact, but becomes exact when you tensor with \( \mathbb{Q} \).

1’. Same with \( k[x, y] \to k(x, y) \).

Hint: The Koszul complex

\[
0 \longrightarrow R \stackrel{(y/x)}{\longrightarrow} R^2 \stackrel{(x, y)}{\longrightarrow} R \longrightarrow 0
\]

is a nice example; understand why it works.

2. Show that \( Z \to \mathbb{Z}[\sqrt{-3}] \) is faithfully flat.

Hint: as a \( \mathbb{Z} \)-module, \( \mathbb{Z}[\sqrt{-3}] \cong \mathbb{Z}^2 \).

2’. Same with \( k[x] \to k[x, y]/(y^2 = x^3 + x) \).

3. If you still have some time, show that if there is a faithfully flat map \( R \to S \) then glob dim \( R \leq \text{glob dim} S \). Problems 1 and 1’ above show that you can’t do without “faithfully.”