Worksheet 15

Math 607, Homological Algebra

Wednesday, May 6, 2020

In Example 17.3, Eisenbud considers the ring

\[ R = k[x, y, z]/(x - 1)z, \]

and explains that \( x, (x - 1)y \) is a regular sequence in that order, but not in the other order \( (x - 1)y, x \), which is irritating. In lecture today we saw that this kind of thing cannot happen in a local ring.

1. In \( k^3 \), sketch the sets \( (x - 1)z = 0 \) and \( x = 0 \) and \( (x - 1)y = 0 \).

2. Consider the maximal ideal \( m = (x, (x - 1)y) = (x, y) \subset R \). Show that in the localization \( R_m \), the sequence \( x, (x - 1)y \) is regular in either order.

3. Let \( n \subset R \) be any other maximal ideal. Show that in \( R_n \), the sequence \( x, (x-1)y \) fails to be regular in any order for the silly reason that it generates the unit ideal.