Let $R = k[x_1, \ldots, x_n]$, let $m = (x_1, \ldots, x_n)$, let $I = (f_1, \ldots, f_\ell) \subset m$, and let $J$ be the Jacobian matrix

$$J = \begin{pmatrix} \frac{\partial f_1}{\partial x_1} & \cdots & \frac{\partial f_\ell}{\partial x_1} \\ \vdots & \ddots & \vdots \\ \frac{\partial f_1}{\partial x_n} & \cdots & \frac{\partial f_\ell}{\partial x_n} \end{pmatrix}.$$ 

Consider the diagram

$$\begin{array}{ccc}
R^\ell & \xrightarrow{J} & R^n \\
(f_1 \ldots f_\ell) \downarrow & & \downarrow (x_1 \ldots x_n) \\
I \xrightarrow{\ell} & & m
\end{array}$$

which doesn’t quite commute: if

$$f_j = (\text{linear term}) + (\text{quadratic term}) + (\text{cubic term}) + \cdots,$$

then from Euler’s theorem we know that

$$\sum x_i \frac{\partial f_j}{\partial x_i} = (\text{linear term}) + 2 \cdot (\text{quadratic term}) + 3 \cdot (\text{cubic term}) + \cdots.$$ 

But convince yourself that if we tensor with $R/m$, then $m$ becomes $m/m^2$ and gets rid of those higher-order terms, yielding a commutative diagram

$$\begin{array}{ccc}
k^\ell & \xrightarrow{J(0)} & k^n \\
I/mI \xrightarrow{\ell} & & m/m^2.
\end{array}$$

(If you’re not used to the fact that $m \otimes R/m \cong m/m^2$, take the exact sequence

$$0 \to m \to R \to R/m \to 0$$

and tensor with $m$. The image of $m \otimes m \to m$ is $m^2 \subset m$.)