1. Let $\alpha$ be a $k$-form on $X \times \mathbb{R}$. For each $t \in \mathbb{R}$ we get a form $\alpha_t$ on $X$ by restricting to $X \times \{t\}$. Convince yourselves that if $\alpha$ is closed then $\alpha_1 - \alpha_0$ is exact.

2. Let $E$ be a vector bundle on $X$, and let $\nabla_0$ and $\nabla_1$ be two connection on $E$. I claim that $c_k(\nabla_0) - c_k(\nabla_1)$ is exact. Read the following argument and discuss any details you’re not sure about:

Consider $X \times \mathbb{R}$, and let

$$p: X \times \mathbb{R} \to X$$

be the projection. On the pull-back bundle $p^*E$ we get two pull-back connections $p^*\nabla_0$ and $p^*\nabla_1$. Their weighted average

$$\nabla := tp^*\nabla_1 + (1-t)p^*\nabla_0$$

is again a connection on $p^*E$. The Chern form $c_k(\nabla)$ is a closed $2k$-forms on $X \times \mathbb{R}$, and its restriction to $t = 0$ or $t = 1$ is $c_k(\nabla_0)$ or $c_k(\nabla_1)$. Thus the $c_k(\nabla_1) - c_k(\nabla_0)$ is exact.