In lecture we defined the standard $n$-simplex

$$\Delta^n = \{(x_0, \ldots, x_n) \in \mathbb{R}^{n+1} : x_i \geq 0, \sum x_i = 1\},$$

the face maps $\varphi_i : \Delta^{n-1} \to \Delta^n$ given by

$$\varphi_i(x_0, \ldots, x_{n-1}) = (x_0, \ldots, x_{i-1}, 0, x_i, \ldots, x_{n-1}),$$

and for an $n$-simplex $\sigma : \Delta^n \to X$, the boundary

$$\partial \sigma = \sum_{i=0}^{n} (-1)^i \sigma \circ \varphi_i \in C_{n-1}(X),$$

which we extended linearly to give a homomorphism

$$C_n(X) \to C_{n-1}(X).$$

1. Verify that if $i < j$ then $\varphi_j \circ \varphi_i = \varphi_i \circ \varphi_{j-1}$.

2. Use this to verify that $\partial \partial \sigma = 0$. 

Worksheet 16

Math 634, Algebraic Topology

Friday, November 13, 2020