Worksheet 3

Math 634, Algebraic Topology

Monday, October 7, 2020

0. Introduce yourself to your colleague. Do they have any pets?

1. Show that homotopy is an equivalence relation on the set of continuous maps $X \to Y$.

2. Suppose that $f_0, f_1 : X \to Y$ and $g_0, g_1 : Y \to Z$ are continuous maps. Show that if $f_0 \simeq f_1$ and $g_0 \simeq g_1$, then $g_0 \circ f_0 \simeq g_1 \circ f_1$.

3. In lecture I asserted that the maps $f_0, f_1 \colon S^2 \to \mathbb{R}^3 \setminus 0$ defined by

$$f_0(x) = 2x + (1, 0, 0)$$
$$f_1(x) = 3x + (0, 0, 1)$$

are homotopic via the "straight-line homotopy"

$$F(x, t) = (1 - t)f_0(x) + tf_1(x).$$

(a) Convince yourself that if you fix $x \in S^2$ and let t vary, then $F(x, t)$ traces out a straight line segment from $f_0(x)$ to $f_1(x)$.

(b) Show that F takes values in $\mathbb{R}^3 \setminus 0$, so it's really a homotopy between f_0 and f_1.