Worksheet 5

Math 634, Algebraic Topology

Friday, October 9, 2020

0. Introduce yourself to your colleague. Can they recommend any good ice-breaker questions that they've used with their students, or come across in other situations?

1. For a space X and a basepoint $x \in X$, we defined $\pi_1(X, x)$ as the set of maps $\gamma\colon I \to X$ with $\gamma(0) = \gamma(1) = x$, up to homotopy rel. endpoints.

   Regard S^1 as the unit circle in the complex plane

   $$S^1 = \{z \in \mathbb{C} : |z| = 1\},$$

   and take $1 \in S^1$ as its basepoint. (Draw a picture.)

   Give a bijection between $\pi_1(X, x)$ and the set of pointed maps

   $$\ell\colon (S^1, 1) \to (X, x)$$

   up to homotopy rel. basepoint.

   Hint: Apply problems 1(b) and 1(c) from the last homework twice: first for the maps, then for the homotopies.

2. If you have more time, work on §1.1 #5 for next Monday's homework. Show that for a space X, the following three conditions are equivalent:

   (a) Every map $S^1 \to X$ is nullhomotopic.

   (b) Every map $S^1 \to X$ extends to a map $D^2 \to X$.

   (c) $\pi_1(X, x) = 0$ for all $x \in X$.

   Hint: Use problem 2 from the last homework.