0. Introduce yourself to your colleague. Can they recommend any good ice-breaker questions that they’ve used with their students, or come across in other situations?

1. For a space $X$ and a basepoint $x \in X$, we defined $\pi_1(X, x)$ as the set of maps $\gamma : I \to X$ with $\gamma(0) = \gamma(1) = x$, up to homotopy rel. endpoints.

Regard $S^1$ as the unit circle in the complex plane

$$S^1 = \{ z \in \mathbb{C} : |z| = 1 \},$$

and take $1 \in S^1$ as its basepoint. (Draw a picture.)

Give a bijection between $\pi_1(X, x)$ and the set of pointed maps $\ell : (S^1, 1) \to (X, x)$

up to homotopy rel. basepoint.

Hint: Apply problems 1(b) and 1(c) from the last homework twice: first for the maps, then for the homotopies.

2. If you have more time, work on §1.1 #5 for next Monday’s homework. Show that for a space $X$, the following three conditions are equivalent:

(a) Every map $S^1 \to X$ is nullhomotopic.
(b) Every map $S^1 \to X$ extends to a map $D^2 \to X$.
(c) $\pi_1(X, x) = 0$ for all $x \in X$.

Hint: Use problem 2 from the last homework.