First Midterm

Name: ______________________________________

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1. Write parametric equations for the line along which the planes $x + 2y + 3z = 1$ and $2x + 5y - z = 2$ intersect.

2. Consider two lines, one passing through $(1, 0, 0)$ and $(0, 1, 0)$ and the other passing through $(0, -1, 0)$ and $(0, 0, 1)$. Find the minimum distance between them, as follows.

   (a) Write parametric equations for both lines, using s as the parameter for the first line and t as the parameter for the second.

   (b) Write a function $f(s, t)$ that gives the distance *squared* between the point on the first line at time s and the point on the second line at time t.

   (c) Find the minimum value of $f(s, t)$.

3. Consider the surface $z = \dfrac{y^2}{1+x^2}$.

(a) Sketch the slices $x = 0$, $x = \pm 1$, and $x = \pm 2$.

(b) Sketch the slices $y = 0$, $y = \pm 1$, and $y = \pm 2$.

(c) Sketch the slice $z = 1$. Hint: multiply through by $1 + x^2$.

(d) Sketch the surface. The slices you drew in parts (a) through (c) should appear in your sketch.

4. (a) In what direction is the function $f(x, y) = x^2 - y^2$ increasing most steeply at the point $(1, 2)$? What is the slope in that direction? What is the slope in the direction $\langle \frac{1}{2}, \frac{\sqrt{3}}{2} \rangle$?

(b) Write an equation for the tangent plane to the surface $y^2 + z^2 = x^3 - x$ at the point $(2, 2, \sqrt{2})$.

5. Consider the curve $\vec{r}(t) = \langle t^2 - 2t + 1, t^2, 2t^2 - 2t \rangle$.

(a) Find $\vec{v}$, $\vec{T}$, a_T, $\vec{a}_{\parallel}$, $\vec{a}_{\perp}$, and a_N at time $t = 1$. Circle your answers. As a sanity check, make sure that $\vec{a}_{\perp}$ is perpendicular to v.

(b) The curve meets the plane $x + y + 2z = 1$ at two points. One is $(1, 0, 0)$; what is the other?

(c) Find the angle between the curve and the plane at those two points. Hint: First find the angle between the velocity vector to the curve and the normal vector to the plane; then think about what this has to do with the angle between the curve and the plane. Further hint: You can do this without a calculator, so the cosines in question must be among 0, $\pm\frac{1}{2}$, $\pm\frac{\sqrt{2}}{2}$, $\pm\frac{\sqrt{3}}{2}$, and ± 1.