Handout on Ordinals

Math 487

September 25, 2013

From lecture today:

- A set x is called an *ordinal* if $\forall y\ (y \in x \to y \subset x)$.
- If x is an ordinal then $x \cup \{x\}$ is also an ordinal, called the *successor* of x. An ordinal that is not a successor is called a *limit ordinal*.
- An ordinal x is called *finite* if it is either 0 or a successor and every ordinal $y \in x$ is either 0 or a successor. Equivalently we could say that every ordinal $y \subset x$ is either 0 or a successor.

The homework for Monday, October 7 is VI.4.10 and the following problem:

(a) Rewrite the formula $x = y \cup \{y\}$ using only $\in$. As a sanity check, your formula should have exactly two free variables, x and y.

(b) Write a formula using only $\in$ that means "x is an ordinal."

(c) Write a formula using only $\in$ that means "x is a finite ordinal." You may use without proof the fact that the following are equivalent:

- There is an ordinal y with $x = y \cup \{y\}$.
- x is an ordinal and there is a set y with $x = y \cup \{y\}$.

This follows from the axiom of foundation, which among other things prohibits circular inclusions like $z \in y \in z$.