Homework 2

Due Monday, January 23, 2011

1. (a) Show that homotopy is an equivalence relation on the set of continuous maps $X \to Y$.

   (b) Suppose that $f_0, f_1 : X \to Y$ and $g_0, g_1 : Y \to Z$. If $f_0 \simeq f_1$ and $g_0 \simeq g_1$, show that $g_0 \circ f_0 \simeq g_1 \circ f_1$.

2. Let $f : S^1 \to X$. Show that the following are equivalent:

   (a) f is nullhomotopic.

   (b) f extends to a map $D^2 \to X$—that is, there is a map $g : D^2 \to X$ such that the restriction $g|_{S^1} = f$.

3. Let X be a topological space. Define $x \sim x'$ if there is a path from x to x'. Convince yourself that $\sim$ is an equivalence relation (but do not turn this in). The equivalence classes of $\sim$ are called the *path components* of X. We define $\pi_0(X) = X/\sim$.

   (a) Show that a map $f : X \to Y$ induces a map $f_* : \pi_0(X) \to \pi_0(Y)$.

   (b) Show that homotopic maps $X \to Y$ induce the same map on π_0.

   (c) Show that a homotopy equivalence induces a bijection on π_0.

4. What is one question you have about last week's lectures?