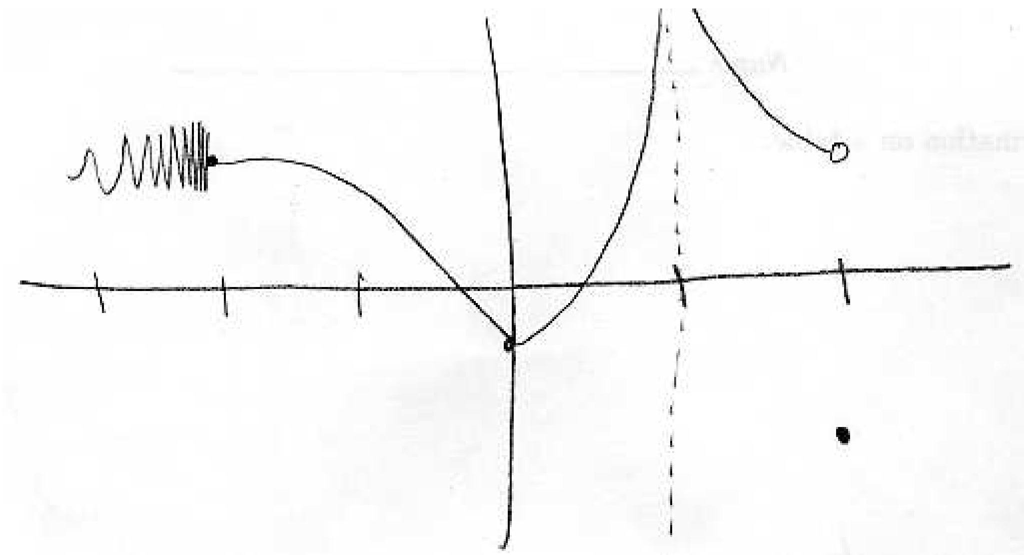
Solutions to Exam 1

1.



2.

$$\lim_{x\to 0}\frac{1-\cos x}{x^2}=\lim_{x\to 0}\frac{(1-\cos x)(1+\cos x)}{x^2(1+\cos x)}=\lim_{x\to 0}\frac{1-(\cos x)^2}{x^2(1+\cos x)}$$
$$=\lim_{x\to 0}\frac{(\sin x)^2}{x^2(1+\cos x)}=\left[\lim_{x\to 0}\frac{\sin x}{x}\right]^2\left[\lim_{x\to 0}\frac{1}{1+\cos x}\right]=1\cdot\frac{1}{2}=\frac{1}{2}$$

3.

$$\begin{aligned}
y &= e^{x/\sqrt{2}}\sin\frac{x}{\sqrt{2}}\\
\frac{dy}{dx} &= \frac{1}{\sqrt{2}}e^{x/\sqrt{2}}\sin\frac{x}{\sqrt{2}}+\frac{1}{\sqrt{2}}e^{x/\sqrt{2}}\cos\frac{x}{\sqrt{2}}\\
\frac{d^2y}{dx^2} &= \frac{1}{2}e^{x/\sqrt{2}}\sin\frac{x}{\sqrt{2}}+\frac{1}{2}e^{x/\sqrt{2}}\cos\frac{x}{\sqrt{2}}+\frac{1}{2}e^{x/\sqrt{2}}\cos\frac{x}{\sqrt{2}}-\frac{1}{2}e^{x/\sqrt{2}}\sin\frac{x}{\sqrt{2}}\\
&= e^{x/\sqrt{2}}\cos\frac{x}{\sqrt{2}}\\
\frac{d^3y}{dx^3} &= \frac{1}{\sqrt{2}}e^{x/\sqrt{2}}\cos\frac{x}{\sqrt{2}}-\frac{1}{\sqrt{2}}e^{x/\sqrt{2}}\sin\frac{x}{\sqrt{2}}\\
\frac{d^4y}{dx^4} &= \frac{1}{2}e^{x/\sqrt{2}}\cos\frac{x}{\sqrt{2}}-\frac{1}{2}e^{x/\sqrt{2}}\sin\frac{x}{\sqrt{2}}-\frac{1}{2}e^{x/\sqrt{2}}\sin\frac{x}{\sqrt{2}}-\frac{1}{2}e^{x/\sqrt{2}}\cos\frac{x}{\sqrt{2}}\\
&= -e^{x/\sqrt{2}}\sin\frac{x}{\sqrt{2}}
\end{aligned}$$

4. Let y be the height of the ball and x be the distance from the wall to the shadow. Then $y = -16t^2 + 64$ and

$$\begin{aligned} \frac{y}{x-20} &= \frac{64}{x} \\ xy &= 64(x-20) \\ \frac{dx}{dt}y + x\frac{dy}{dt} &= 64\frac{dx}{dt} \end{aligned}$$

After 1 second, $y = -16 + 64 = 48$, $x = 80$, and $\frac{dy}{dt} = -32t = -32$, so $\frac{dx}{dt} = -160$ ft/s.