Problem Set 11

Friday, April 21

I. Problems to be graded on completion.

1. Sketch the region bounded by the given curves. Then find the volume of the solid generated by revolving the region around the x- and y-axes. Use either the disc method or the shell method.
   a. $2x + y = 4$, $x = 0$, $y = 0$.
   b. $y = x^2$, $y = x^{1/3}$.
   c. $y = \sin x$, $x = \pi/4$, $x = \pi/2$, $y = 0$. Around the x-axis, you may need to know that $(\sin x)^2 = \dfrac{1 - \cos 2x}{2}$. Around the y-axis, you can't evaluate the integral, so just set it up.
2. The base of a solid is the circle $x^2 + y^2 = 4$. Find the volume of the solid if the cross sections perpedicular to the x-axis are
   a. squares;
   b. equilateral triangles;
   c. triangles of height 3.

- §6.4 #8, 9

II. Problems to be graded on correctness.

1. Sketch the region bounded by $y = \sqrt{r^2 - x^2}$, $x = h$, and $y = 0$, where r is a positive constant and h is a constant between 0 and r. Find the volume of the solid generated by revolving the region around the x-axis.

- §6.2 #27. If you copy from the solution manual, I will be able to tell.

2. Find the length of the curve $y = \dfrac{e^x + e^{-x}}{2}$ between $x = -1$ and $x = 1$. This curve is called the *catenary* and describes the shape of a power line (or any other cable) suspended between two points.
3. A guy in your dorm took calculus last year and thinks he knows it all, but you see through this when you ask for his help on some integrals and he claims that the following things are true. Explain his error in each case, and show definitively that he is wrong by taking the derivative of the right-hand side. (If you live off-campus, use your imagination.)
   a. $\displaystyle\int \frac{dx}{x^3} = \int x^{-3}\, dx = \frac{x^{-4}}{-4} + C.$
   b. $\displaystyle\int x^2 (\sec x)^2\, dx = \frac{x^3}{3} \tan x + C.$
   c. $\displaystyle\int \cos(x^3 + 1)\, dx = \sin(x^3 + 1) \left(\frac{x^4}{4} + x \right) + C.$
   d. $\displaystyle\int \frac{dx}{4x^2 + 1} = \log |4x^2 + 1| + C.$