

# Problem Set 12

Friday, April 28

## I. Problems to be graded on completion.

1. Determine the average value of the each function on the indicated interval and find a point in this interval at which the function takes on this average value:
  - a.  $f(x) = x^2$  on  $[-1, 1]$
  - b.  $f(x) = x^3$  on  $[-1, 1]$
  - c.  $f(x) = x^{-1}$  on  $[1, 4]$
  - d.  $f(x) = \sin x$  on  $[0, \pi]$
  - e.  $f(x) = \sqrt{x}$  on  $[1, 9]$
- §6.5 #2, 4, 10, 12, 19, 21. For the tank problems, just pump the water to the top of the tank, not 5 feet above as the directions say to.

## II. Problems to be graded on correctness.

1. A rod 6 meters long is placed on the  $x$ -axis between  $x = 0$  and  $x = 6$ . The density of the rod is  $12/\sqrt{x+1}$  kilograms per meter. Find the mass of the rod and center of mass.
  - a. Sketch the graph of a function  $f(x)$ . You don't need to write down a formula, just draw something with a few wiggles. In particular, don't choose something dumb like  $f(x) = x$ .
  - b. Sketch the graph of  $f(x+c)$  where  $f$  is the function you made up in part (a) and  $c$  is some positive number.
  - c. Draw vertical lines  $x = a$  and  $x = b$  on your graph from part (b). It would be nice if  $a < b$ .  
Shade the area that represents  $\int_a^b f(x+c) dx$ .
  - d. Draw vertical lines  $x = a+c$  and  $x = b+c$  on your graph from part (a). Shade the area that represents  $\int_{a+c}^{b+c} f(x) dx$ .
  - e. Using  $u$ -substitution, show that the integrals in parts (c) and (d) are equal.
  - f. Describe in words what the substitution  $u = x + c$  does.