Problem Set 2

Tuesday, January 31

I. Problems to be graded on completion.

- Find the following limits:
  1. $\displaystyle\lim_{x\to 0}\frac{\sin 4x}{\sin 2x}$.
  2. $\displaystyle\lim_{x\to 0}\frac{\sin x^2}{x}$. Note that $\sin x^2$ means $\sin(x^2)$, which is different from $(\sin x)^2$.
  3. $\displaystyle\lim_{x\to 0}\frac{(\sin 3x)^2}{5x^2}$.
  4. $\displaystyle\lim_{x\to \frac{\pi}{3}}\frac{\sin x}{x}$.
- §2.8 #12, 14, 16, 18 (look at the hint for 17).
- §2.8 #21, 23, 25, 29, 50
- §7.3 #4, 6, 8, 10. What we call $\log x$ the book calls $\ln x$.
- Find $\displaystyle\lim_{x\to\infty}\Big(\log(x+1)-\log x\Big)$.

II. Problems to be graded on correctness.

1. Find $\lim_{x \to 0} \dfrac{\tan 3x}{2x^2 + 5x}$.
2. Find $\lim_{x \to \infty} \dfrac{\sin x}{x}$.
3. Show that $\lim_{h \to 0^+} (1 + hx)^{1/h} = e^x$.
4. Write

$$2 \log x - 4 \log \frac{1}{y} - 3 \log(xy)$$

   as one logarithm (a) by moving everything inside and cancelling multiplicatively and (b) by moving everything outside and cancelling additively. The two answers should agree.
5. Recall that a *polynomial* is something like $-3x^4 + \frac{1}{12}x^3 - x + 2$. The *degree* of the polynomial is the highest power of x that appears in it. The *leading coefficient* is the number that multiplies that highest power of x. For example, $-3x^4 + \frac{1}{12}x^3 - x + 2$ is a polynomial of degree 4 with leading coefficient -3. $x^5 - 6x + 3$ is a polynomial of degree 5 with leading coefficient 1. A *rational function* is a quotient of two polynomials, for example $\dfrac{-3x^4 + \frac{1}{12}x^3 - x + 2}{x^5 - 6x + 3}$.
   a. Write down a polynomial $p(x)$ of degree at least 3, a different polynomial $q(x)$ of the same degree, and a third polynomial $r(x)$ of a higher degree. Make all the leading coefficients different from 1.
   b. Find $\lim_{x \to \infty} \dfrac{p(x)}{r(x)}$.
   c. Find $\lim_{x \to \infty} \dfrac{p(x)}{q(x)}$.
   d. Find $\lim_{x \to \infty} \dfrac{r(x)}{q(x)}$.
   e. What can you say in general about the limit as x goes to infinity of a rational function, in terms of the degrees and leading coefficients of the numerator and the denominator? What about the limit as $x \to -\infty$? Would it be hard to prove your statement? Write in complete sentences and be clear.