Problem Set 2
Tuesday, January 31

I. Problems to be graded on completion.

• Find the following limits:

1. \( \lim_{x \to 0} \frac{\sin 4x}{\sin 2x} \).

2. \( \lim_{x \to 0} \frac{\sin x^2}{x} \). Note that \( \sin x^2 \) means \( \sin(x^2) \), which is different from \( (\sin x)^2 \).

3. \( \lim_{x \to 0} \frac{(\sin 3x)^2}{5x^2} \).

4. \( \lim_{x \to \frac{\pi}{3}} \frac{\sin x}{x} \).

• §2.8 #12, 14, 16, 18 (look at the hint for 17).

• §2.8 #21, 23, 25, 29, 50

• §7.3 #4, 6, 8, 10. What we call \( \log x \) the book calls \( \ln x \).

• Find \( \lim_{x \to \infty} \left( \log(x + 1) - \log x \right) \).
II. Problems to be graded on correctness.

1. Find \( \lim_{x \to 0} \tan 3x \div 2x^2 + 5x \).

2. Find \( \lim_{x \to \infty} \frac{\sin x}{x} \).

3. Show that \( \lim_{h \to 0^+} (1 + hx)^{1/h} = e^x \).

4. Write 

\[
2 \log x - 4 \log \frac{1}{y} - 3 \log(xy)
\]

as one logarithm (a) by moving everything inside and cancelling multiplicatively and (b) by moving everything outside and cancelling additively. The two answers should agree.

5. Recall that a polynomial is something like \(-3x^4 + \frac{1}{12}x^3 - x + 2\). The degree of the polynomial is the highest power of \(x\) that appears in it. The leading coefficient is the number that multiplies that highest power of \(x\). For example, \(-3x^4 + \frac{1}{12}x^3 - x + 2\) is a polynomial of degree 4 with leading coefficient \(-3\). \(x^5 - 6x + 3\) is a polynomial of degree 5 with leading coefficient 1. A rational function is a quotient of two polynomials, for example \(-3x^4 + \frac{1}{12}x^3 - x + 2 \div x^5 - 6x + 3\).

a. Write down a polynomial \(p(x)\) of degree at least 3, a different polynomial \(q(x)\) of the same degree, and a third polynomial \(r(x)\) of a higher degree. Make all the leading coefficients different from 1.

b. Find \( \lim_{x \to \infty} \frac{p(x)}{r(x)} \).

c. Find \( \lim_{x \to \infty} \frac{p(x)}{q(x)} \).

d. Find \( \lim_{x \to \infty} \frac{r(x)}{q(x)} \).

e. What can you say in general about the limit as \(x\) goes to infinity of a rational function, in terms of the degrees and leading coefficients of the numerator and the denominator? What about the limit as \(x \to -\infty\)? Would it be hard to prove your statement? Write in complete sentences and be clear.