# Problem Set 4

Thursday, February 16

**I. Problems to be graded on completion.**

1. Evaluate:
   a. $\dfrac{d}{dx}\dfrac{x}{1+x^2}$.
   b. $\dfrac{d}{dx}\log(\sin x)$.
   c. $\dfrac{dy}{dx}$ if $y = (x^{3/2}+1)(2x^2+3x)$.
   d. $f''(x)$ if $f(x) = \sin x$.
   e. $\dfrac{d^2}{dx^2}(x^4 + 3x^{-1} + 5)$.
   f. $\dfrac{d^2y}{dx^2}$ if $y = \sec x$.
2. Use the tangent line approximation $f(x+h) \approx f(x) + f'(x)h$ to approximate $\sin(32^\circ)$. Remember that our formula $\frac{d}{dx}\sin x = \cos x$ is only true when $x$ is in radians.
3. Recall the Newton-Raphson method for approximating the roots (i.e. zeros) of a function $f(x)$: if $x_1$ is your first guess, then you can get better approximations $x_2, x_3, \ldots$ as follows:

$$x_2 = x_1 - \frac{f(x_1)}{f'(x_1)} \qquad x_3 = x_2 - \frac{f(x_2)}{f'(x_2)} \qquad \cdots$$

   If $f(x) = x^2 - 24$ and your first guess is $x_1 = 5$, find $x_4$.

- §3.7 # 36

4. An object projected vertically upward from ground level returns to earth in 8 seconds. What was the initial velocity in feet per second?
5. A ball is thrown straight up from ground level. How high will the ball go if it reaches a height of 64 feet in 2 seconds?

- §3.9 # 2, 4, 6

**II. Problems to be graded on correctness.**

1. Find
$$\frac{d}{dx}\left[f(3/x)\frac{d}{dx}g(x^4-5x)\right]$$
in terms of the derivatives of $f$ and $g$.
2. Draw a picture that explains how the Newton-Raphson method works.
3. a. Use the tangent line approximation
$$f(x+h) \approx f(x) + f'(x)h$$
to approximate the cube root of the number in Feynman's anecdote. Do not use a calculator. Do not feel that you need to come up with a decimal answer; a fraction will do. $1.03 = \frac{103}{100}$.
   b. Using a calculator, find the exact value. How accurate was your approximation?
4. Recall the product rule: $[f(x)g(x)]' = f'(x)g(x) + f(x)g'(x)$, or, more briefly, $(fg)' = f'g + fg'$.
   a. Find $(fg)''$ (take the derivative of $f'g + fg'$).
   b. Find $(fg)'''$ (take the derivative of your answer in part (a)).
   c. Find the fourth derivative $(fg)^{(4)}$ (take the derivative of your answer in part (b)).
   d. Do you recognize these coefficients? Without using the product rule again, guess what $(fg)^{(7)}$ will be.
5. In the park there are two intersect paths, one going north-south and the other going east-west. At a certain time, Bill is 40 feet north of the intersection and is walking south at 4 ft/sec, and Hillary is 30 feet east of the intersection and is walking east at 5 ft/sec. How fast is the straight-line distance between them changing at that time? Is it increasing or decreasing?