# Solutions to Problem Set 4

**I. Problems to be graded on completion.**

1. a. $(1+x^2)^{-1} + x[-(1+x^2)^{-2}]2x = (1+x^2)^{-1} - 2x^2(1+x^2)^{-2} = \dfrac{1-x^2}{(1+x^2)^2}$.

   b. $\dfrac{1}{\sin x}\cos x = \cot x$.

   c. $(\frac{3}{2}x^{1/2})(2x^2+3x) + (x^{3/2}+1)(4x+3)$.

   d. $-\sin x$.

   e. $12x^2 + 6x^{-3}$.

   f. $(\sec x \tan x)\tan x + \sec x(\sec x)^2 = \sec x(\tan x)^2 + (\sec x)^3 = 2(\sec x)^3 - \sec x$.

2. $\sin(32°) = \sin(30° + 2°) = \sin(\frac{\pi}{6} + \frac{\pi}{90}) \approx \sin(\frac{\pi}{6}) + \cos(\frac{\pi}{6})\frac{\pi}{90} = \frac{1}{2} = \frac{\sqrt{3}}{2}\frac{\pi}{90} = \frac{1}{2} + \frac{\pi}{60\sqrt{3}} \approx$ 0.530229989403904. The exact value is 0.529919264233205, so this approximation is correct to within .06%.

3. Observe that $f'(x) = 2x$.

$$
\begin{aligned}
x_1 &= 5\\
x_2 &= 4 - \frac{1}{10} = \frac{49}{10} = 4.9\\
x_3 &= \frac{49}{10} - \frac{\frac{1}{100}}{\frac{98}{10}} = \frac{4801}{980} \approx 4.898979591836735\\
x_4 &= \frac{4801}{980} - \frac{\frac{1}{960400}}{\frac{4801}{490}} = \frac{46099201}{9409960} \approx 4.898979485566358.
\end{aligned}
$$

   ${x_4}^2$ differs from 24 by one part in 88547347201600. The exact value is 4.898979485566356.

36. $v = s' = v_0 - 32t$. When $t = 3$, $v = -140$, so $v_0 = -44$. When $t = 0$, $s = 0$, and when $t = 3$, $s = -276$, so the cliff is 276 feet tall.

4. The object's height is described by $y = -16t^2 + v_0 t$. When $t = 8$, $y = 0$, so $v_0 = 128$.

5. Again, $y = -16t^2 + v_0 t$. When $t = 2$, $y = 64$, so $v_0 = 64$. Moreover, $v = y' = -32t + v_0 = -32t + 64$, so when the ball reaches its maximum height, i.e. when $v = 0$, $t = 2$, so $y = 64$.

2. The volume of a sphere is $V = \frac{4}{3}\pi r^3$, so $\frac{dV}{dt} = 4\pi r^2 \frac{dr}{dt}$. At all times, $\frac{dV}{dt} = 3$, so when $r = 3$, $\frac{dr}{dt} = \frac{1}{12\pi} \approx .0265$ inches per second.

4. The volume of a cone is $V = \frac{1}{3}\pi r^2 h$, and for this cone, $\frac{r}{h} = \frac{3}{10}$, so $r = \frac{3}{10}h$, so $V = \frac{3\pi}{100}h^3$, so $\frac{dV}{dt} = \frac{9\pi}{100}h^2\frac{dh}{dt}$. At all times, $\frac{dV}{dt} = 3$, so when $h = 5$, $\frac{dh}{dt} = \frac{4}{3\pi} \approx .424$ centimeters per second.

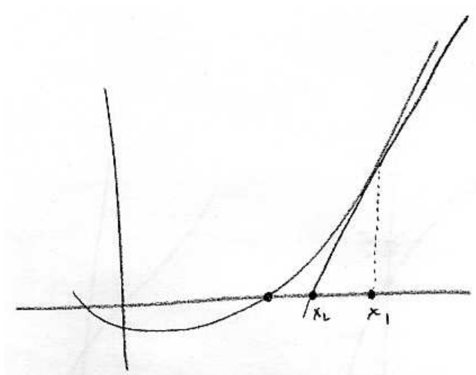
6. Let $r$ be the length of rope and $x$ be the distance from the boat to the dock. Then $x^2 + 10^2 = r^2$, so $2x\frac{dx}{dt} = 2r\frac{dr}{dt}$. At all times, $\frac{dr}{dt} = 2$, and when $r = 25$, $x = 5\sqrt{21}$, so $\frac{dx}{dt} = \frac{10}{\sqrt{21}} \approx 2.18$ feet per second.

**II. Problems to be graded on correctness.**

1.

$$f'(3x^{-1})(-3x^{-2})g'(x^4-5x)(4x^3-5)+f(3x^{-1})\left[g''(x^4-5x)(4x^3-5)^2+g'(x^4-5x)(12x)\right]$$

2.




3. a. $f(x)=\sqrt[3]{x}=x^{1/3}$, so $f'(x)=\frac{1}{3}x^{-2/3}=\dfrac{1}{3\sqrt[3]{x}^2}$. From the story, $12^3=1728$, so

$$\sqrt[3]{1729.03}=\sqrt[3]{1728+1.03}\approx\sqrt[3]{1728}+\frac{1}{3\sqrt[3]{1728}^2}1.03=12+\frac{1}{3\cdot 144}\frac{103}{100}=12+\frac{103}{43200}.$$

b. $\sqrt[3]{1729.03}=12.00238378569172$, where as $12+\frac{103}{43200}\approx 12.00238425925926$, so our approximation is correct to about one part 25 million.

4. Recall the product rule: $[f(x)g(x)]'=f'(x)g(x)+f(x)g'(x)$, or, more briefly, $(fg)'=f'g+fg'$.

a. $(f'g+fg')'=(f''g+f'g')+(f'g'+fg'')=\mathbf{f''g+2f'g'+fg''}$.

b. $(f''g+2f'g'+fg'')'=(f'''g+f''g')+2(f''g'+f'g'')+(f'g''+fg''')=\mathbf{f'''g+3f''g'+3f'g''+fg'''}$.

c. $(f'''g+3f''g'+3f'g''+fg''')'=(f^{(4)}g+f'''g')+3(f'''g'+f''g'')+3(f''g''+f'g''')+(f'g'''+fg^{(4)})=\mathbf{f^{(4)}g+4f'''g'+6f''g''+4f'g'''+fg^{(4)}}$.

d. $f^{(7)}g+7f^{(6)}g'+21f^{(5)}g''+35f^{(4)}g'''+35f'''g^{(4)}+21f''g^{(5)}+7f'g^{(6)}+fg^{(7)}$.

5. Let $y$ be Bill's position, $x$ Hillary's, and $r$ the distance between them. Then $x^2+y^2=r^2$, so when $y=40$ and $x=30$, $r=50$.

$$\begin{aligned}x^2+y^2&=r^2\\2x\frac{dx}{dt}+2y\frac{dy}{dt}&=2r\frac{dr}{dt}\\x\frac{dx}{dt}+y\frac{dy}{dt}&=r\frac{dr}{dt}\\40(-4)+30(5)&=50\frac{dr}{dt}\\\frac{dr}{dt}&=-\frac{1}{5}\end{aligned}$$

so the distance between them is decreasing at one fifth of a foot per second.