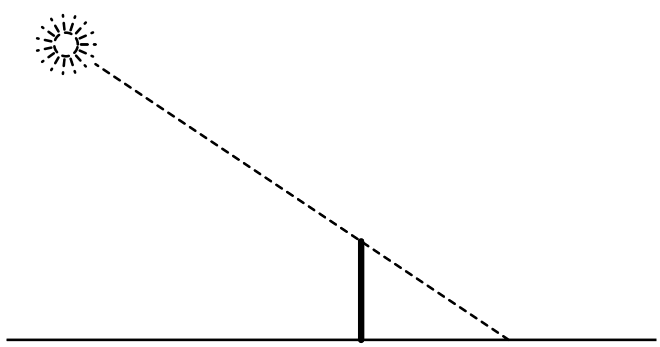
Problem Set 5

Wednesday, February 22

I. Problems to be graded on completion.

- §3.8 #14, 16, 18, 37, 38
- §3.9 #22. You may wish to use the law of cosines: $c^2 = a^2 + b^2 - 2ab\cos\theta$.

1. A rotating beacon is located 2 miles out in the water. Let A be the point on the shore that is closest to the beacon. As the beacon rotates at 10 rev/min, the beam sweeps down the shore once each time it revolves. Assume that the shore is straight. How fast is the point where the beam hits the shore moving at an instant when the beam is lighting up a point 2 miles downshore from the point A?
2. As the sun sets, the shadow of a 25-meter-high wall is growing longer. How fast is the shadow lengthening when it is 50 meters long? Hint: How long does it take the sun to go around the Earth? How many radians is that? Consequently, what is $\frac{d\theta}{dt}$?



- §7.1 #28, 30
- §7.2 #16, 20, 30, 38, 40

II. Problems to be graded on correctness.

1. Suppose that $(x-y)^2 - y = 0$.

   a. Solve for y. Hint: $y = \dfrac{(2x+1) \pm \sqrt{(2x+1)^2 - 4x^2}}{2}$. But you have to show this, and you have to simplify.

   b. Using your answer to part (a), find $\dfrac{dy}{dx}$.

   c. Using the original equation, differentiate implicitly and solve for $\dfrac{dy}{dx}$ in terms of x and y.

   d. Substitute your answer to part (a) for y in your answer to part (c). You may wish to find $2(x-y)$ or $2x-2y$ and then substitute that so you don't have to make the same simplification twice.

   e. Show your answers to parts (b) and (d) agree.

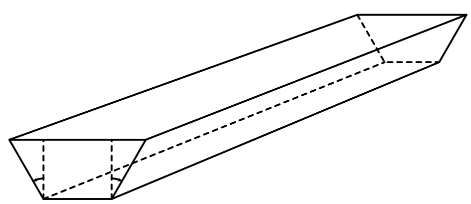
2. Let $y = \dfrac{(x^2+1)^2}{4\sqrt[3]{3x+2}}$.

   a. Find y' the usual way.

   b. Find y' using logarithmic differentiation.

   c. Show that your answers to parts (a) and (b) agree.

3. The trough shown below is constructed by fastening together three slabs of wood of dimensions 10 ft $\times$ 1 ft, and then attaching the construction to a wooden wall at each end. The angle θ was originally 30°, but because of poor construction the sides are collapsing. The trough is full of water. At what rate (in ft^3/sec) is the water spilling out over the top of the trough if the sides have each fallen to an angle of 45°, and are collapsing at the rate of 1° per second?



4. Recall the product rule: if $y = u_1(x)u_2(x)$ then $y' = u_1'(x)u_2(x) + u_1(x)u_2'(x)$, or, more briefly, if $y = u_1u_2$ then $y' = u_1'u_2 + u_1u_2'$. Here $u_1(x)$ and $u_2(x)$ are just two different functions, like $u(x)$ and $v(x)$ or $f(x)$ and $g(x)$, but we will run out of letters if we call them u and v.

   a. If $y = u_1u_2u_3$, find y'. Hint: Think if $u_1u_2u_3$ as u_1 times u_2u_3.

   b. If $y = u_1u_2u_3u_4$, find y'.

   c. If $y = u_1u_2 \cdots u_n$ (i.e., n functions multiplied together), what will y' be? Prove your answer using logarithmic differentiation. If the dot-dot-dot… notation seems uncouth to you, write sentences. But try not to be longwinded.