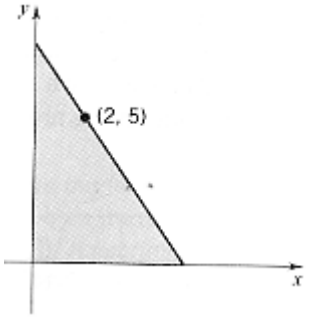
Problem Set 6

Wednesday, March 8

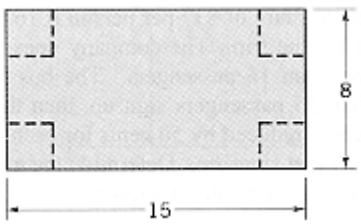
I. Problems to be graded on completion.

- §4.1 #25, 31
- §4.4 #11
- §4.5 #18
- §4.7 #46

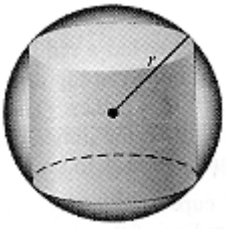
1. A triangle is formed by the coordinate axes and a line through the point $(2, 5)$ as shown below. Determine the slope of the line that minimizes the area of the triangle.



2. You are going to turn a rectangular piece of cardboard into a box by cutting squares out of the corners a folding up the sides. If the dimensions of the original piece are 8×15, what size squares should you cut out to maximize the volume? What is the maximum possible volume?



3. A cylinder is inscribed in a sphere of radius R as shown below. Find the dimensions of the cylinder that maximize its volume.



- §9.1 #4, 6, 8, 12
- §9.2 #2, 8, 28, 32

II. Problems to be graded on correctness.

1. A power line is needed to connect a power plant on the shore of a (straight) river to an island 4 kilometers downstream and 1 kilometer offshore.
   a. Find the minimum cost for such a line given that it costs \$50,000/km to lay wire underwater and \$30,000/km to lay wire underground. Also find the point where the line should leave the shore.
   b. Same questions, but with the costs underwater and underground reversed. The answer is intuitively obvious, but what happens to the calculus solution?
   c. If it costs \$30,000/km to lay wire underground and the minimum cost is achieved by having the line leave the shore 2 kilometers from the power plant, how much does it cost to lay wire underwater?
2. Write a max/min problem based on another course you are taking, your job, or some other aspect of your life. If you work together with other students, you must all come up with different problems. If your problem is really good, I may use it on the next exam.
3. Solve the problem you wrote for #2.
4. Let f be a differentiable function and $M \geq 0$ be a number. Show that:
   a. If $|f(x) - f(y)| \leq M|x - y|$ for all x and y then $|f'(x)| \leq M$ for all x. Hint: Look for the difference quotient.
   b. If $|f'(x)| \leq M$ for all x then $|f(x) - f(y)| \leq M|x - y|$ for all x and y. Hint: Suppose there are two numbers x and y for which this fails and use the mean value theorem to produce a contradiction.
5. a. Try to evaluate $\lim_{h \to 0} \dfrac{e^{-1/h^2}}{h}$ using L'Hôpital's rule and describe what happens.
   b. Write the limit in the equivalent form $\lim_{h \to 0} \dfrac{1/h}{e^{1/h^2}}$ and evaluate it using use L'Hôpital's rule.
   c. Define a function $f(x)$ by
   $$f(x) = \begin{cases} e^{-1/x^2} & \text{if } x \neq 0 \\ 0 & \text{if } x = 0 \end{cases}.$$
   Using the definition of the derivative, show that f is differentiable at 0. What is $f'(0)$?