1. Let $h$ be the height of the rectangle and $r$ be the radius of the circle, which is half the width of the rectangle, as below:

The perimeter is $\pi r + 2h + 2r = (\pi + 2)r + 2h$. Since this is constrained to be 24, $h = 12 - (\frac{\pi}{2} + 1)r$. The area is

\[
A = \frac{1}{2}\pi r^2 + 2rh = \frac{\pi}{2}r^2 + 2r[12 - (\frac{\pi}{2} + 1)r] = \frac{\pi}{2}r^2 + 24r - (\frac{\pi}{2} + 2)r^2
\]

We wish to choose $r$ to maximize $A$, so we set $0 = \frac{dA}{dr} = 24 - (\pi + 4)r$, so $r = \frac{24}{\pi + 4} \approx 3.36$, so after a little arithmetic, we find that $h = \frac{24}{\pi + 4}$ as well. That is, the rectangle should be a twice as wide as it is tall.

2. Let $t$ be the time in hours and $x(t)$ be the driver’s position. By the mean value theorem there must be a time $c$ between 0 and 2 for which

\[v(c) = x'(c) = \frac{x(2:00) - x(12:00)}{2:00 - 12:00} = \frac{150 \text{ mi}}{2 \text{ hr}} = 75 \text{ mi/hr}\]

so he must have been speeding at some point.

3. Consider $\log(e^x + 3x)^{1/x} = \frac{1}{x} \log(e^x + 3x) = \frac{\log(e^x + 3x)}{x}$. As $x \to 0$, this is of the form $\frac{0}{0}$, so we can use L’Hôpital’s rule:

\[
\lim_{x \to 0} \log(e^x + 3x)^{1/x} = \lim_{x \to 0} \frac{\log(e^x + 3x)}{x} = \lim_{x \to 0} \frac{\frac{1}{e^x + 3x}(e^x + 3)}{1} = 4.
\]

Thus $\lim_{x \to 0} (e^x + 3x)^{1/x} = \lim_{x \to 0} e^{\log(e^x + 3x)^{1/x}} = e^{\lim_{x \to 0} \log(e^x + 3x)^{1/x}} = e^4 \approx 54.6$. 


4. \( y = (\sin x)^3 \), so \( y' = 3(\sin x)^2 \cos x \), so

\[
y'' = 6 \sin x (\cos x)^2 - 3(\sin x)^3
= 3 \sin x [2(\cos x)^2 - (\sin x)^2] \\
= 3 \sin x [2(1 - (\sin x)^2) - (\sin x)^2] \\
= 3 \sin x [2 - 3(\sin x)^2].
\]

When \( y = 0 \), \( \sin x = 0 \), so \( x = 0, \pi, \) or \( 2\pi \). When \( y' = 0 \), either \( \sin x = 0 \), so \( x = 0, \pi, \) or \( 2\pi \); or \( \cos x = 0 \), so \( x = \pi/2 \) or \( 3\pi/2 \). When \( y'' = 0 \), either \( \sin x = 0 \), so \( x = 0, \pi, \) or \( 2\pi \); or \( 2 = 3(\sin x)^2 \), so \( \sin x = \pm \sqrt{2/3} \), so \( x = .96, 2.19, 4.10, \) or \( 5.33 \). The signs of \( y \), \( y' \), and \( y'' \) are as follows:

When \( x = \pi/2 \), \( y = 1 \). When \( x = 3\pi/2 \), \( y = -1 \). When \( x = .96 \) or \( 2.19 \), \( y = (2/3)^{3/2} \approx .54 \). When \( x = 4.10 \) or \( 5.33 \), \( y = -(2/3)^{3/2} \approx -.54 \).
5. $y = \cos x$ looks like this,

![Graph of $y = \cos x$]

so $y = (\sec x)^2 = \frac{1}{(\cos x)^2}$ will be 1 when $\cos x$ is 1 and go to infinity when $\cos x$ goes to 0:

![Graph of $y = (\sec x)^2$]

First we find the intersection. If $(\sec x)^2 = 2$ then $\sec x = \sqrt{2}$ (since we’re working on between $-\pi/2$ and $\pi/2$ we can ignore the negative square root), so $\cos x = \frac{1}{\sqrt{2}} = \frac{\sqrt{2}}{2}$, so $x = \pi/4$ or $-\pi/4$. Now

$$\int_{-\pi/4}^{\pi/4} [2 - (\sec x)^2] \, dx = \left[ 2x - \tan x \right]_{-\pi/4}^{\pi/4} = \left( \frac{\pi}{2} - 1 \right) - \left( -\frac{\pi}{2} + 1 \right) = \pi - 2 \approx 1.14.$$