Worksheet 5

February 6, 2008

1. Let $f(x, y) = x^2 y + \cos(\pi x y)$. Show that

$$\frac{\partial^2 f}{\partial x\, \partial y} = \frac{\partial^2 f}{\partial y\, \partial x}.$$

2. Consider again the surface $z = y^2 - x^2$.

   (a) Find $\dfrac{\partial z}{\partial x}$ and $\dfrac{\partial z}{\partial y}$.

   (b) Evaulate these at the point $(1, 2)$.

   (c) The plane $x = 1$ intersects our surface in a curve. Find the equation of the tangent line to this curve.

   (d) Do the same for the plane $y = 2$.

   (e) Find the equation of the plane containing the lines you found in parts (c) and (d). Solve for z. Do you see the numbers from part (b)? Is this a coincidence?

   (f) Sketch the surface, the two curves, the two tangent lines, and the plane.