Worksheet 6

February 11, 2008

1. In §14.3 #67, you are asked to show that

$$f(x, y, z) = \frac{1}{\sqrt{x^2 + y^2 + z^2}}$$

satisfies Laplace's equation

$$\frac{\partial^2 f}{\partial x^2} + \frac{\partial^2 f}{\partial y^2} + \frac{\partial^2 f}{\partial z^2} = 0.$$

   (a) Do it.

   (b) Do it more slickly by writing $f^{-2} = x^2 + y^2 + z^2$ and differentiating implicitly.

2. Let $f(x, y) = \dfrac{2xy}{x^2 + y^2}$.

   (a) What is the domain of this function?

   (b) Sketch slices $y = -1$, $y = 0$, $y = 1$, and $y = 2$. Observe that they are all continuous. Argue that all the x-slices are continuous as well.

   (c) Consider the line $y = x$ in the xy-plane. Describe the portion of the function's graph lying over this line.

   (d) What about the line $y = -x$? $y = 2x$? More generally, the line in the xy-plane that makes an angle θ with the x-axis?

   (e) Is it possible to "fill in the whole" and assign a value to $f(0, 0)$ to make the function continuous?

   (f) Understand the picture on page 979.