Worksheet 12

March 12, 2008

1. Sketch the solid whose volume is given by

\[ \int_0^2 \int_0^2 (8 - x^2 - y^2) \, dx \, dy. \]

Find its volume by evaluating the integral.

2. (a) Evaluate

\[ \int_0^1 \int_0^1 \frac{x - y}{(x + y)^3} \, dx \, dy \]

by partial fractions.

(b) Evaluate

\[ \int_0^1 \int_0^1 \frac{x - y}{(x + y)^3} \, dy \, dx \]

(c) How is this possible?

(d) What can you say about

\[ \iint_R \frac{x - y}{(x + y)^3} \, dA, \]

where \( R \) is the region \( 0 \leq x \leq 1, 0 \leq y \leq 1 \)?

3. Continue to work on the gutter problem from Monday.