Worksheet 15

April 2, 2008

1. Sketch the regions whose areas are given by

\[ \int_{-1}^{0} \int_{0}^{\sqrt{1-x^2}} dy \, dx \quad \int_{-1}^{1} \int_{0}^{\sqrt{1-x^2}} dy \, dx \quad \int_{0}^{1} \int_{0}^{\sqrt{1-x^2}} dy \, dx \]

\[ \int_{-1}^{0} \int_{-\sqrt{1-x^2}}^{0} dy \, dx \quad \int_{-1}^{1} \int_{-\sqrt{1-x^2}}^{0} dy \, dx \quad \int_{0}^{1} \int_{-\sqrt{1-x^2}}^{0} dy \, dx \]

\[ \int_{-1}^{0} \int_{0}^{0} dy \, dx \quad \int_{-1}^{1} \int_{-\sqrt{1-x^2}}^{0} dy \, dx \quad \int_{0}^{1} \int_{-\sqrt{1-x^2}}^{0} dy \, dx \]

2. Last Monday you integrated \( xy \) over (a) the portion of the unit disc lying in the first quadrant, (b) the portion of the unit disc lying in the upper half-plane, and (c) the whole unit disc. Do the same in polar coordinates. Explain the symmetry by graphing \( \frac{1}{2} \sin 2\theta \).

3. Integrate \( \frac{1}{1+x^2+y^2} \) over the whole plane

(a) in rectangular coordinates, and

(b) in polar coordinates.

4. Find the centroid of a cone.

5. Find the volume of the intersection of two balls.