(This worksheet requires a planimeter.) We will refer to this diagram:

1. (a) Let the base be at the origin and the viewer at \((x, y)\). Express \(x\) and \(y\) in terms of \(a, b, \theta,\) and \(\phi\).
(b) To find the area of the region whose boundary the viewer traces, you could integrate \(\frac{1}{2}(x\,dy - y\,dx)\). Express this differential form in terms of \(a, b, \theta, \phi, d\theta,\) and \(d\phi\). Simplify your answer using angle addition formulas.

2. (a) Now suppose the wheel is at \((x, y)\) and is moved by a tiny amount \(\langle dx, dy \rangle\). How far does it roll? (If it moves parallel to the arm, it will not roll. If it moves perpendicular to the arm, it will roll a lot. Thus you should dot with a unit vector perpendicular to the arm.)
(b) Express \(x\) and \(y\) in terms of \(b, c, \theta,\) and \(\phi\).
(c) Express your answer to part (a) in terms of \(b, c, \theta, \phi, d\theta,\) and \(d\phi\).

3. (a) Argue that \(\oint c\,d\phi = 0\) if the base is not in the middle of your region.
(b) Similarly, argue that \(\oint \cos(\theta + \phi)\,d(\theta + \phi) = 0\).
(c) Compare the distance the wheel rolls to the area of the region whose boundary the viewer traces.