Worksheet 21

April 30, 2008

1. (a) Sketch the part of the surface \( z = x^2 + y^2 \) inside the cylinder \( x^2 + y^2 = 1 \). Find its area.
   
   (b) Sketch the part of the surface \( z = x^2 - y^2 \) inside the cylinder \( x^2 + y^2 = 1 \). Find its area.

2. (a) Find the center of mass of half of a solid ball.
   
   (b) Find the center of mass of half of a sphere (just the surface).

3. Find the moment of inertia of a sphere (just the surface, not the interior) of radius \( R \) about the \( z \)-axis. Compare this to the moment of inertia of a solid ball about the \( z \)-axis, which you computed on the exam.

4. You will find yourself integrating over a sphere pretty often. For a sphere of radius \( R \), what is \( d\sigma \) in rectangular coordinates? In polar coordinates?

5. Sketch the ellipsoid

\[
\frac{x^2}{a^2} + \frac{y^2}{a^2} + \frac{z^2}{b^2} = 1
\]

twice, once if \( a < b \) (called a “prolate spheroid”) and then if \( a > b \) (called an “oblate spheroid”). Find its surface area.