1. [30 points] Find each of the following derivatives:
   
   (a) For \( f(x) = 4x^{3/2} + \frac{5}{x^2} \)
   
   \[
   \frac{df}{dx}(x) = 4 \frac{d}{dx}(x^{3/2}) + 5 \frac{d}{dx}(x^{-2}) = 6x^{1/2} - \frac{10}{x^3}.
   \]

   (b) For \( y = x^{3}\sqrt{5x + 3} \)
   
   \[
   \frac{dy}{dx} = (5x + 3)^{1/2} \frac{d}{dx}(x^{3}) + x^{3} \frac{d}{dx}(5x + 3)^{1/2} = 3x^{2}(5x + 3)^{1/2} + \frac{5}{2}x^{3}(5x + 3)^{-1/2}
   \]

2. [20 points] Find \( \frac{dv}{dt} \) given that \( u \) and \( v \) are differentiable functions of \( t \) satisfying
   
   \[
   u^2 - 2uv + v^3 = 14, \quad \text{with} \quad u(2) = 3, \quad v(2) = -1, \quad \text{and} \quad \frac{du}{dt} \bigg|_{t=2} = 5.
   \]

   \[
   \frac{d}{dt}(u^2 - 2uv + v^3) = 2u \frac{du}{dt} - 2v \frac{du}{dt} - 2u \frac{dv}{dt} + 3v^2 \frac{dv}{dt} = 0
   \]

   \[
   2(3)(5) - 2(-1)(5) - 2(3) \frac{dv}{dt} + 3(-1)^2 \frac{dv}{dt} = 0
   \]

   \[
   40 - 3 \frac{dv}{dt} = 0
   \]

   so

   \[
   \frac{dv}{dt} = \frac{40}{3}.
   \]

3. [5 points] Find the instantaneous rate of change of the function \( s = \ln \frac{t^2}{t^6 + 1} \) when \( t = 1 \).

   \[
   s = \ln \frac{t^2}{t^6 + 1} = 2 \ln t - \ln(t^6 + 1);
   \]

   \[
   \frac{ds}{dt} = \frac{2}{t} - \frac{6t^5}{t^6 + 1} \quad \text{so} \quad \frac{ds}{dt} \bigg|_{t=1} = \frac{2}{1} - \frac{6}{2} = -1.
   \]

4. [20 points] Find all relative maxima and minima of the function \( f(x) = x^3 - 3x^2 \).

   \[
   f'(x) = 3x^2 - 6x = 3x(x - 2), \quad \text{so} \quad f'(x) = 0 \iff x = 0, 2.
   \]

   But

   \[
   \begin{array}{c|cccc}
   x & -2 & 0 & 2 & 4 \\
   f(x) & -20 & 0 & -4 & 16 \\
   \end{array}
   \]

   so \((0, 0)\) is a relative maximum and \((2, -4)\) is a relative minimum.
5. **5 points** Calculate the limit: \( \lim_{h \to 0} \frac{\ln 3(4 + h) - \ln 12}{h} = \)

\[
\lim_{h \to 0} \frac{\ln 3(4 + h) - \ln 12}{h} = \frac{d}{dx} \ln 3x \bigg|_{x=4} = \frac{1}{x} \bigg|_{x=4} = \frac{1}{4}.
\]

6. **10 points** We must fence a rectangular field with one side of the field along an existing wall. For the three remaining sides it costs $10 per foot for the side opposite the wall but only $4 per foot along the other two sides. If we need 720 square feet for the field, find the dimensions that will provide the least cost.

The objective function is cost and the constraint is area. So if we let \( y \) be the length of the side opposite the wall and \( x \) the length of the other two sides, then

\[
C = 2(4x) + 10y \quad \text{and} \quad A = xy = 720.
\]

So

\[
C(x) = \frac{7200}{x} + 8x \quad \text{and} \quad C'(x) = -\frac{7200}{x^2} + 8 = 0 \iff x^2 = 900 \iff x = \pm 30.
\]

But \( x = -30 \) makes no sense, and clearly the cost can be arbitrarily large for \( x \) close to 0 or real long. So the minimum cost is when \( x = 30 \) feet and \( y = 720/x = 24 \) feet.

7. **10 points** The demand for a certain business jet is

\[ q = 12e^{-p/3} \]

where \( q \) is the number of jets sold annually when the price is \( p \) million dollars.

(a) Find the marginal revenue annually when the price is $2 million for each jet.

The revenue as a function of price is

\[ R(p) = pq = 12pe^{-p/3} \quad \text{so} \quad R'(p) = 12e^{-p/3} - 4pe^{-p/3} = 4(3 - p)e^{-p/3}. \]

Then the marginal revenue when \( p = 2 \) is

\[
\left. \frac{dR}{dp} \right|_{t=2} = 4e^{-2/3} = 2.05 \text{ jets/million dollars}.
\]

(b) Determine the price \( p \) at which the revenue from jet sales is maximum, and find the number sold at this price.

\[
\frac{dR}{dp} = 0 \iff p = 3.
\]

So \( p = 3 \) million dollars is the price for maximum revenue, and for that price the number sold is \( q = 12e^{-1} = 4.4 \) jets per year.

(The price \( p = 3 \) gives maximum revenue since \( p = 3 \) is the only critical point and \( R(0) = 0 \) and \( R(100) \) is very small.)