Solutions to selected homework problems. 1.

1.4.22. This gives an equation $4.98x+5.98y = 100.64$, or $498x+598y = 10064$. Reduce by 2: $249x + 299y = 5032$. Now apply Euclidian algorithm to 249 and 299: $299 = 249+50$, $249 = 4\cdot 50+49$, $50 = 49+1$, so

$$1 = 50 - 49 = 5 \cdot 50 - 249 = 5 \cdot 299 - 6 \cdot 249.$$

The general solution of our equation is $x = x_0 + 299t$, $y = y_0 - 249t$, where (x_0, y_0) is a particular solution. One way to get a particular solution is to set $x_0 = -6\cdot 5032$, $y_0 = 5\cdot 5032$. To work with a bit smaller numbers we can observe that $5032 = 20 \cdot 249 + 52 = 19 \cdot 249 + 299 + 2$. Thus, we can get a solution of the form $x_0 = 19 - 6 \cdot 2 = 7$, $y_0 = 1+5\cdot 2 = 11$. The general solution of the linear equation is $x = 7+299t$, $y = 11 - 249t$. Since we want x and y to be positive, we have to set $t = 0$. The answer: $x = 7$, $y = 11$.

1.4.36. Set $d = (a, b)$. Then the condition $a|bc$ is equivalent to $\frac{a}{d}|\frac{b}{d}c$. Since $(\frac{a}{d}, \frac{b}{d}) = 1$ (by Theorem 1.14), Theorem 1.13 implies that $\frac{a}{d}|c$. Hence, $a|dc$ as required.

1.4.37. Suppose f is a common positive divisor of $\frac{a}{d}$ and $\frac{b}{d}$, so $f|\frac{a}{d}$ and $f|\frac{b}{d}$. This implies that $fd|a$ and $fd|b$. Thus, fd is a common divisor of a and b. Since d is the greatest common divisor, it follows that $fd \leq d$, so $f \leq 1$, which implies that $f = 1$.

1.4.38. By Theorem 1.15, the general solution of this linear equation has form $x = x_0+\frac{b}{d}t$, $y = y_0-\frac{a}{d}t$, where (x_0, y_0) is a particular solution. Since a and b have opposite signs, if we choose t to be of the same sign as b and with very large $|t|$ we will get positive x and y.

1.5.28. $49 \equiv 3(\mod 23)$, hence, $49^4 \equiv 3^4 \equiv 81 \equiv 12(\mod 23)$.

1.5.32. $50 \equiv -1(\mod 17)$, hence, $50^9 9 \equiv -1 \equiv 16(\mod 17)$.

1.5.42. This is not true: take $c = 2$, $b = 1$, $a = -1$.

1.5.46. We have $a^n b^n \equiv (ab)^n \equiv 1 \equiv a^n(\mod m)$. Since $(a^n, m) = 1$, by Theorem 1.18, we get $b^n \equiv 1(\mod m)$.

4.1.14. This is not a complete solution: $x \equiv 11(\mod 12)$ is missing.

4.1.16. This is a complete solution.