Mark each statement True or False. Justify each answer.
(If true, cite appropriate facts or theorems. If false, explain why or give a counterexample that shows why the statement is not true in every case.) In parts (a)–(f), \( v_1, \ldots, v_p \) are vectors in a nonzero finite-dimensional vector space \( V \), and \( S = \{v_1, \ldots, v_p\} \).

a. The set of all linear combinations of \( v_1, \ldots, v_p \) is a vector space.
b. If \( \{v_1, \ldots, v_{p-1}\} \) spans \( V \), then \( S \) spans \( V \).
c. If \( \{v_1, \ldots, v_{p-1}\} \) is linearly independent, then so is \( S \).
d. If \( S \) is linearly independent, then \( S \) is a basis for \( V \).
e. If \( \text{Span} \, S = V \), then some subset of \( S \) is a basis for \( V \).
f. If \( \dim V = p \) and \( \text{Span} \, S = V \), then \( S \) cannot be linearly dependent.
g. A plane in \( \mathbb{R}^3 \) is a two-dimensional subspace.
h. The nonpivot columns of a matrix are always linearly dependent.
i. Row operations on a matrix \( A \) can change the linear dependence relations among the rows of \( A \).
j. Row operations on a matrix can change the null space.
k. The rank of a matrix equals the number of nonzero rows.
l. If an \( m \times n \) matrix \( A \) is row equivalent to an echelon matrix \( U \) and if \( U \) has \( k \) nonzero rows, then the dimension of the solution space of \( Ax = 0 \) is \( m - k \).
m. If \( B \) is obtained from a matrix \( A \) by several elementary row operations, then \( \text{rank} \, B = \text{rank} \, A \).
n. The nonzero rows of a matrix \( A \) form a basis for \( \text{Row} \, A \).
o. If matrices \( A \) and \( B \) have the same reduced echelon form, then \( \text{Row} \, A = \text{Row} \, B \).
p. If \( H \) is a subspace of \( \mathbb{R}^3 \), then there is a \( 3 \times 3 \) matrix \( A \) such that \( H = \text{Col} \, A \).
q. If \( A \) is \( m \times n \) and \( \text{rank} \, A = m \), then the linear transformation \( x \mapsto Ax \) is one-to-one.
r. If \( A \) is \( m \times n \) and the linear transformation \( x \mapsto Ax \) is onto, then \( \text{rank} \, A = m \).