STUDY GUIDE FOR THE FINAL EXAM

The final examination focuses on the main concepts and topics of Sections 6.1-6.7, and 7.1-7.3. There may be a few definitions on the exam. The most important definitions include: orthogonal basis, eigenvector, eigenvalue, least-squares solution, QR factorization.

A number of questions will require that you give reasons for your answers, which often involve a reference to a theorem such as: Best Approximation Theorem, Orthogonal Decomposition Theorem, Orthogonal Diagonalization Theorem, Principal Axes Theorem.

Definitions:
- Length of a vector, orthogonal vectors, orthogonal set, orthogonal basis.
- Orthogonal projection of \( y \) onto a line through \( 0 \), orthogonal projection of \( y \) onto a subspace \( W \), orthogonal complement of \( W \).
- QR factorization.
- Least-squares solution, normal equations.
- Design matrix, parameter vector, observation vector.
- Symmetric matrix, orthogonally diagonalizable.

Theorems:
- Chapter 6: Theorem 2 (Pythagorean Theorem) – Know proof, Theorem 4.
- Theorem 8 (The Orthogonal Decomposition Theorem).
- Theorem 9 (The Best Approximation Theorem).
- Theorem 12 (QR Factorization), Theorem 13 (Normal Equations).
- Theorem 15. Know basic calculation for proof of Theorem 14.
- Chapter 7: Theorems 1, 2 (Orthogonal Diagonalization), 3, 4 (Principal Axes Theorem), 5 (Quadratic Forms and Eigenvalues), 6, 7.

Important skills:
- Compute length of vector, distance between vectors.
- Determine if a set is orthogonal, normalize a vector.
- Construct an orthonormal set from an orthogonal set. Know \( ||x||^2 = x^T x = x \cdot x \).
- Check a set for orthogonality.
- Compute orthogonal projection onto a line (through \( 0 \)) or a subspace.
- Decompose a vector into a component in the direction of \( u \) and a component orthogonal to \( u \). Decompose a vector into the sum of a vector in a subspace \( W \) and a vector in \( W^\perp \).
- Find the vector in a subspace \( W \) that is closest to a specified vector.
- Find the distance from a subspace to a specified vector.
- Perform the Gram-Schmidt process on a linearly independent set of vectors.
- Compute the QR factorization of a matrix with linearly independent columns.
- Find a least-squares solution of \( Ax = b \). Compute the least-squares error.
- Orthogonally diagonalize a symmetric matrix \( A \).
- Classify a given quadratic form as positive definite, negative definite, indefinite, positive semidefinite, or negative semidefinite.
- Find the maximum and the minimum values of a quadratic form subject to \( x^T x = 1 \).

Applications:
- Construct a QR factorization of a matrix, use a QR factorization to produce a least-squares solution of \( Ax = b \).
- Find the least-squares line to fit a set of data, set up a design matrix for a least-squares fit to data by a specified equation, identify the parameter vector and the observation vector.
- Write the normal equations for the linear model \( y = X\beta + \epsilon \) (namely, \( X^T X = X^T y \)).