Math 342, Spring 2021

STUDY GUIDE FOR MIDTERM

The midterm exam focuses on the main concepts and topics of Sections 5.1-5.7. There may be a few definitions on the exam. The most important definitions include:

- Eigenvalue, eigenvector, eigenspace, diagonalization.
- A number of questions will require that you give reasons for your answers. These reasons will often involve a reference to a theorem.

**Definitions:**

- Eigenvalue, eigenvector, eigenspace, diagonalizable matrices.
- Similar matrices.
- Matrix of a linear transformation $T$ relative to a basis $B$, $[T]_B$.

**Theorems:**

- Chapter 5: Theorems 1, 2, 4, 5 (the diagonalization theorem), 6, and 8.

**Important skills:**

- Find a change-of-coordinates matrix, use this matrix to find a coordinate vector
- Determine if a number (vector) is an eigenvalue (eigenvector) of a matrix
- Find the characteristic equation and eigenvalues of a $2 \times 2$ matrix. Find the eigenvalues of a triangular matrix, listed according to their multiplicities.
- Find a basis for an eigenspace.
- If $A$ is diagonalizable, find $P$ and $D$ such that $A = PDP^{-1}$. Show how to compute high powers of a diagonalizable matrix.
- Find the $B$-matrix $[T]_B$ of a linear transformation $T : V \to V$ relative to a basis $B$ of $V$.
- Verify statements involving similarity of matrices.
- Find complex eigenvalues and corresponding eigenvectors.
- Find a factorization of a $2 \times 2$ matrix with a complex eigenvalue, $A = PCP^{-1}$, where the transformation $x \mapsto Cx$ is a composition of a rotation and possibly a scaling transformation. Determine the angle of the rotation and the scale factor.
- Find the solution of a difference equation $x_{k+1} = Ax_k$ in terms of the eigenvectors of $A$, and describe the discrete evolution of the dynamical system. Use eigenvectors to describe the directions of greatest attraction and greatest repulsion. Be able to classify the origin as an attractor, a repellor, or a saddle point. Describe how a change of variable can decouple a system of difference equations.
- Same for the differential equation $\mathbf{x}' = A\mathbf{x}$.