Math 253 (Calc III), Spring 2019
Approximate List of Topics:

- **Sequences** Convergence and divergence - what they mean. How many terms before you get within a certain accuracy.
- The mathematical definition of a limit, as well as what it means for the limit to be infinity.
- Different ways to write down sequences (explicit, recursive, etc)
- Testing for convergence: squeeze theorem, monotone sequence theorem, geometric sequences
- Finding limits: plugging in continuous functions, extension to a function and L'Hopital's rule.
- Limits of recursive sequences.
- **Series** Convergence and divergence - what they mean. The difference between a sequence and a series.
- Geometric series and p-series.
- The divergence test. The ratio test.
- Series with positive entries: the integral test, the comparison test.
- Series with positive and negative entries: definition of absolute convergence, the absolute convergence test, the alternating series test.
- Remainder estimates for geometric series, the integral test, the alternating series test.
- **Power Series** What they are, and how they behave at the center.
- Finding the radius of convergence (using the ratio test). Finding the interval of convergence (using other tests on the boundary).
- Manipulation of power series: addition, multiplication, derivatives, integrals, substituting λx^k.
- Reindexing of power series (sometimes needed enable addition).
- Power series which are geometric, and their manipulations.
- Using power series and series bounding methods to estimate values and integrals.
- **Taylor Series** Understanding what it means to approximate a function around a point c to degree k. Knowing what the coefficients of the Taylor series tell you about the function.
- Computing derivatives to compute the Taylor series around any point, or to compute the k-th order approximation.

- The Taylor series at 0 for common functions: $\frac{1}{1-x}$, $\ln(x+1)$, $\sin x$, $\cos x$, e^x, $\arctan(x)$. Using these series to estimate a number (say, $\ln(2)$) to a desired accuracy.
- Taylor's remainder theorem!!!!!!!! (Also called the Taylor inequality.) Using this to bound the error on the degree k approximation at a given point.
- **Ordinary Differential Equations** What an ODE looks like. What an initial value is.
- Finding the first few derivatives of a particular solution at a given point using an ODE, and using this to compute Taylor polynomials.
- Finding recursive formulas for the general solution to an ODE, centered at a given point. Using these recursive formulas to compute Taylor polynomials.

Topics in your book or in my mind, but NOT covered in the final: (These were either purposely left out of the curriculum, poorly covered in class, or lost to the snowstorm)

- Telescoping sums.
- The limit comparison test. (This one is useful, by the way, I just preferred to make you suffer by thinking through things.)
- Comparative remainder estimates for the comparison test, the absolute convergence test.
- The binomial theorem, that is, the Taylor series centered at zero for $(1+x)^k$ for all values of k.
- Recognizing the common Taylor series (like e^x, $\sin x$) when evaluated, i.e. when x is set to a specific value.
- Using the Taylor remainder theorem to find the degree of a Taylor polynomial which computes the true value of a function at a given point to within a given accuracy. (This is an important application.)
- Using the Taylor remainder theorem to find the interval on which a given Taylor polynomial computes the true value of a function to within a given accuracy.
- Using the Taylor remainder theorem to prove that a Taylor series converges to a function (when it does).
- Applications to science. Approximating real world processes with Taylor series.
- Solving more difficult ODEs with power series (the ODEs which require recentering, multiplication, and other manipulation techniques). In theory you know enough to do this, but just need a lot of examples. (Aside: perhaps the main point of power series techniques is that they let you approximate the solutions to ODEs that AREN'T easily solvable by other means. Sadly, most of the problems I can currently give you are solvable using the techniques in MAT256.)