Approximate List of Topics:

- **Sequences** Convergence and divergence - what they mean.
- The mathematical definition of a limit. How many terms before you get within a certain accuracy.
- Different ways to write down sequences (explicit, recursive, etc)
- Testing for convergence: squeeze theorem, monotone sequence theorem, geometric sequences
- Finding limits: plugging in continuous functions, extension to a function and L'Hopital's rule.
- Limits of recursive sequences.
- **Series** Convergence and divergence - what they mean. The difference between a sequence and a series.
- Geometric series and p-series.
- The divergence test. The ratio test. The integral test.
- Remainder estimates for geometric series and the integral test.

General notes:

1. This practice test is purposely a bit too long, to give a little more practice.
2. Pace yourself. Make sure you try all the problems.
3. If you need more space, use the back of the previous page.
4. "Why" does something converge or diverge? Answer this with the name of a test you can apply, or some other reason. If this test requires you to do something, do it.

   For example, if you identify a geometric series, you should indicate what a and r are. If you use the squeeze theorem, you should indicate which other sequences you are squeezing between, and why they converge.

1. Does the sequence converge or diverge? WHY? If it converges, what is the limit.

   (a)
$$a_n = \sqrt{\frac{4n^2 - 2n + 3}{2n^2 + 17}}$$

   (b)
$$b_n = \frac{n^n}{n!}$$

   (c)
$$c_n = (\frac{1}{2}, \frac{-1}{2}, \frac{1}{3}, \frac{-2}{3}, \frac{1}{4}, \frac{-3}{4}, \frac{1}{5}, \frac{-4}{5}, \ldots)$$

   (d)
$$d_n = 8000(\frac{99}{100})^n$$

   (e)
$$e_n = \frac{\ln n}{\ln(\ln n)}$$

2. Consider the sequence $a_n = \frac{n^2-2}{n^2}$.

   (a) How many terms of the sequence are needed before it gets within $.01$ of the limit? Within $.001$ of the limit?

   (b) Using the mathematical definition of limit, prove that the sequence converges.

3. Consider the sequence $a_1 = 10$ and $a_{n+1} = \frac{1}{4}a_n + 3$. Here are three problems to do, but you should choose what order to do them in.

   - Prove using induction that the sequence is bounded below.
   - What else should you prove to deduce that the sequence is convergent? You don't need to prove it.
   - What is the limit?

4. Does the series converge or diverge? WHY?

(a)

$$\sum_{n=1}^{\infty} \frac{n10^n}{13^n}$$

(b) (Hint: what is the derivative of $\ln(\ln(n))$?)

$$\sum_{n=2}^{\infty} \frac{1}{n \ln n}$$

(c)

$$\sum_{n=1}^{\infty} \frac{5^n}{n!}$$

(d)

$$\frac{3}{7} + \frac{4}{8} + \frac{5}{9} + \frac{6}{10} + \dots$$

(e)

$$\sum_{n=3}^{\infty} \frac{1}{n^{1.05}}$$

(f)

$$\sum_{n=1}^{\infty} (\frac{1}{n^{1.05}} + n^{1.05})$$

5. What is the sum of the following series?

$$\sum_{n=6}^{\infty}(\frac{6}{7})^n$$

6. How many terms of the series $\sum_{n=1}^{\infty} \frac{4}{n^5}$ must you take to approximate the sum of the series to within 10^{-8}?