THIS TIME I DECIDED TO WRITE A LOT OF EXTRA PROBLEMS TO GIVE MORE PRACTICE. The actual midterm will have about 6 problems. If you want to practice something with approximately the same length as the midterm, maybe try: problems 1, 3abc, 4a, 6, 9ab, 11. For another sample midterm, maybe problems: 3de, 4b, 5, 7, 12, 13bc, 14.

General notes:

1. Pace yourself. Make sure you write something on every problem to get partial credit.
2. If you need more space, use the back of the previous page.
3. "Why" does something converge or diverge? Answer this with the name of a test you can apply, or some other reason. If this test requires you to do something, do it.

   For example, if you identify a geometric series, you should indicate what a and r are. If you use the squeeze theorem, you should indicate which other sequences you are squeezing between, and why they converge.
4. General test-taking trick: divide and conquer. If a problem is testing two things, and you know how to do one, do that part for partial credit. Indicate the different parts of the problem and show that you know what you do and don't know. (Don't waste time writing down an outline if you actually know what to do, it should be clear from your work, you'll get that partial credit anyway if the work is wrong.)
5. For example: You're supposed to compute $(1+x)\arctan x$ as a power series. Are you stuck because you don't remember the power series for $\arctan$? Make a note and substitute a power series you do remember, like $\ln(1+x)$. Then at least you can demonstrate some of the other techniques needed for the problem for more partial credit. Warning: The place this can backfire is for an error estimation problem, where you unwittingly replace a sequence which is easy to estimate (e.g. an alternating series) with one which is harder.
6. For example: You know that $\sum \frac{n}{n^2+3}$ diverges because it looks like $\sum \frac{1}{n}$, and you know you're supposed to use the comparison test to handle this, but you don't know how. Say so.

Approximate List of Topics:

- **Series** The comparison test. The alternating series test. The absolute value test.
- Remainder estimates for the comparison test, the alternating series test, and the absolute value test.
- **Power Series** What they are, and how they behave at the center.
- Finding the radius of convergence (using the ratio test). Finding the interval of convergence (using other tests on the boundary).
- Manipulation of power series: addition, multiplication, derivatives, integrals, substituting λx^k.
- **Taylor Series** Understanding what it means to approximate a function around a point c to degree k. Knowing what the coefficients of the Taylor series tell you about the function.
- The Taylor series at 0 for common functions: $\frac{1}{1-x}$, $\ln(x+1)$, $\sin x$, $\cos x$, e^x, $\arctan(x)$. Recognizing these series when x is set to a specific value. Using these series to estimate a number (say, $\ln(2)$) to a desired accuracy.
- Computing derivatives to compute the Taylor series around any point, or to compute the degree k Taylor polynomial.
- One use of Taylor's remainder theorem: given N and d, bound the error on the Taylor remainder $R_N(x)$ for x within distance d of the center. (We will discuss more uses of Taylor's remainder theorem in class soon, such as proving that a Taylor series converges to a function; these will not be on the midterm.)

1. Consider the series
$$\frac{1}{4} - \frac{1}{8} + \frac{1}{12} - \frac{1}{16} + \ldots$$

   (a) Is the series convergent? WHY?

   (b) How many terms of the sum must one take in order to be within .1 of the limit?

   (c) Is the series absolutely convergent? WHY?

2. What does the **ratio test** say about the following series?

   (a) $\displaystyle\sum_{n=1}^{\infty}(-1)^n \frac{2^n}{n!}$

   (b) $\displaystyle\sum_{n=1}^{\infty} \frac{n^2+3}{n^3+2}$

3. Does the series converge or diverge? WHY?

(a)

$$\sum_{n=1}^{\infty} \frac{n7^n}{3^{2n+5}}$$

(b)

$$\sum_{n=0}^{\infty} \frac{n^2+2}{n^3+3}$$

(c)

$$-1 - \frac{1}{4} + \frac{1}{9} + \frac{1}{16} - \frac{1}{25} - \frac{1}{36} + \frac{1}{49} + \ldots$$

(d)

$$\sum_{n=6}^{\infty} (-1)^n \frac{2n^2+2}{n^3+3}$$

(e)

$$\sum_{n=2}^{\infty} (-1)^n \frac{n^5-3}{2n^5+3n^3+n-1}$$

4. Find the interval of convergence of the following power series.

   (a) $\displaystyle\sum_{n=0}^{\infty} \frac{(x-2)^n}{n3^n}$

   (b) $\displaystyle\sum_{n=0}^{\infty} \frac{4^n(x+9)^n}{n^3+1}$

5. Compute $\int_0^{1/10} \arctan(t^2)dt$ to within 10^{-9}.

6. Find a power series centered at zero for the following functions. (Note: I could also ask for the radius of convergence.)

   (a) $\dfrac{1}{4-3x}$

   (b) $\displaystyle\int_0^x \frac{1}{1+t^6}\,dt$

   (c) The derivative of $\displaystyle\sum_{n=0}^{\infty} \frac{2^n(n!)x^n}{(2n)!}$.

7. Find a power series centered at zero for the following functions. Write out the first three nonzero terms explicitly. (Note: I could also ask for the radius of convergence.)

   (a) e^{x^3}

   (b) (Hint: what is the derivative of $\frac{1}{1-x}$?) $\dfrac{1}{(1+2x)^2}$

   (c) $\ln(1-x^3)$

8. Find $\cos(.5)$ to within $\frac{1}{500}$.

9. Using any method, find the first few terms of the Taylor series, up to the cubic term (i.e. the x^3 term).

   (a) $e^x \cos x$ centered at 0.

   (b) $\ln(x)$ centered at 2.

   (c) e^{3x} centered at -5.

10. Find the degree three Taylor polynomial $T_3(x)$ for $\frac{1}{1-x}$ centered at 5. Bound the error on the interval $[4, 6]$.

11. Find the degree 5 Taylor polynomial $T_5(x)$ for $3\sin x$ centered at 0. What error bound does the Taylor Remainder Theorem give on the interval $[-.2, .2]$?

12. Let $f(x) = e^x \sin x$. For your convenience, we have calculated the first several derivatives of f.

$$f'(x) = e^x(\sin x + \cos x)$$
$$f''(x) = 2e^x \cos x$$
$$f'''(x) = 2e^x(\cos x - \sin x)$$
$$f''''(x) = -4e^x \sin x$$

Compute the degree 3 Taylor polynomial $T_3(x)$ for $f(x)$ centered at 0, and bound the error on the interval $[-2, 2]$.

13. Is this series convergent or divergent? If it is convergent, what is the sum?

(a) $\frac{1}{2} - \frac{1}{2^2 \cdot 2} + \frac{1}{2^3 \cdot 3} - \frac{1}{2^4 \cdot 4} + \ldots$

(b) $3 - \frac{3^3}{3} + \frac{3^5}{5} - \frac{3^7}{7} + \ldots$

(c) $3 - \frac{3^3}{3!} + \frac{3^5}{5!} - \frac{3^7}{7!} + \ldots$

14. Find the first few terms of a power series centered at 0 for the following function, up to the x^3 term.

$$(x^2 - 5)\left(\sum_{n=0}^{\infty} (n+1)x^n\right).$$