Math 316 (Fund. of analysis), Winter 2018
Quiz 1
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Name:

General notes:

1. Terminology:
   - Supremum = least upper bound.
   - Injective = 1-to-1 = into.
   - Surjective = onto.

2. Justification: On an exam, by default, you should justify your statements. If an example or counterexample is the best justification, you should provide one. Otherwise, you should provide a brief conceptual explanation (one or two sentences).
   - To justify that “Every closed interval contains 0” is false, you should provide a counterexample, such as $[1, 2]$.
   - To justify that “Some closed interval contains 0” is true, you should provide an example, such as $[-1, 1]$.
   - To justify that “Every closed interval contains some nonzero number” is true, you should explain. “For any interval $[a, b]$, either $a \neq 0$ or $b \neq 0$.”

3. If I want a complete proof, I will say “Prove that ...”

4. If I don’t need any justification, I will say “no justification necessary.” No justification is needed for a definition.

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1. **(6 pts)** Let $A$ be a subset of $\mathbb{R}$. Give a formal definition of the *infimum* or *greatest lower bound* of $A$.

2. **(6 pts)** Consider the set

$$B = \left\{ 3 - \frac{1}{n} \mid n \in \mathbb{N} \right\} \cup (497.500)$$

inside $\mathbb{R}$. Does $B$ have more than one supremum? Compute every supremum of $B$ (no justification necessary).

3. **(8 pts)** Suppose $C$ and $D$ are nonempty subsets of $\mathbb{R}$ which are bounded above, but not bounded below. Define the set

$$C \cdot D = \{ c \cdot d \mid c \in C, d \in D \}.$$

Does $\sup(C \cdot D)$ exist? If so, find a formula for it. If not, find a counterexample.
4. **(20 pts)** For each of the following statements, is it true or false? Justification is required.

   (a) Any open interval in \( \mathbb{R} \) contains an irrational number.
   (b) Every nonempty subset of \( \mathbb{R} \) has a supremum.
   (c) There is no collection of open intervals \( I_n \) such that \( \bigcap_{n \in \mathbb{N}} I_n \) is a closed interval.
   (d) The set \( \{ \text{even numbers} \} \cup \{ \text{multiples of 7} \} \) is countable.

5. **(10 pts)** Let \( a < b \) be real numbers, and consider the set \( T = \mathbb{Q} \cap [a, b] \). Prove that \( \sup T = b \).