Math 317 (Fund. of analysis), Spring 2018
Quiz 1

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Name:

Q1		9
Q2		11
Q3		15
Q4		15
Total		50

General notes:

1. If you run out of space, use the other side of the paper!
2. Terminology: Supremum = least upper bound. Injective = 1-to-1 = into. Surjective = onto.
3. Justification: On an exam, by default, you should justify your statements. If an example or counterexample is the best justification, you should provide one. Otherwise, you should provide a brief conceptual explanation (**one or two sentences**).
   - To justify that "Every closed interval contains 0" is false, you should provide a counterexample, such as $[1, 2]$.
   - To justify that "Some closed interval contains 0" is true, you should provide an example, such as $[-1, 1]$.
   - To justify that "Every closed interval contains some nonzero number" is true, you should explain. "For any interval $[a, b]$, either $a \neq 0$ or $b \neq 0$."
4. If I want a complete proof, I will say "Prove that ..."
5. If I don't need any justification, I will say "no justification necessary." No justification is needed for a definition.
6. Make sure to specify any terms which are not in the statement of the problem. For example, when defining the term *continuous*, don't say "f is continuous if..." say "A function $f : \mathbb{R} \to \mathbb{R}$ is continuous if..."

1. **(9 pts)** For the function $f\colon \mathbb{R} \to \mathbb{R}$, $f(x) = x^3$, prove that $f'(5) = 75$. (You may use the formula $x^3 - y^3 = (x-y)(x^2 + xy + y^2)$.)

2. **(11 pts)** (a) Define the term *compact*.

   (b) Let $A \subset \mathbb{R}$ be closed and bounded. Prove that A is compact.

3. **(15 pts)** (a) Let $A \subset \mathbb{R}$, and $f\colon A \to \mathbb{R}$. Define what it means for f to be *uniformly continuous*.

   (b) Briefly justify the fact that $f(x) = \frac{1}{x}$ is uniformly continuous on $[1, 2]$.

   (c) Briefly justify the fact that $\frac{1}{x}$ is not uniformly continuous on $(0, 1]$.

4. **(15 pts)** For each of the following statements, is it true or false? Justification is required.

   (a) If f is continuous on $[a, c]$ then there is some $b \in (a, c)$ for which $f'(b) = \frac{f(c)-f(a)}{c-a}$.

   (b) If f is differentiable on (a, b), and $f'(x) \neq 0$ for any $x \in (a, b)$, then either f is increasing or f is decreasing.

   (c) If C is the Cantor set and $f\colon C \to \mathbb{R}$ is continuous, then f attains a maximum value.