Outline of Bump Ch 6-8

Point of Ch 6: A derivation of an algebra is a map $X: A \to A$ s.t.
$X(fg) = X(f)g + fX(g)$. They form a lie algebra, i.e. $[X, Y]$ is a derivation. 

• Derivations of $C^\infty(M)$ are called vector fields.

Can show that if $f$ vanishes in a nbhd of $x$, then so does $Xf$ for any derivation.

So descends to $C^\infty(M) / V_x = \{f | f = 0$ in nbhd of $x\}$

$\text{Der}(C^\infty(M)) \to \text{V}_x$ gen's at $x$.

• Derivations of $\text{V}_x \equiv$ tangent vectors at $x$. They form a V.S. of dim $N$,
spacer by \[ \frac{d}{dx} \] for any local chart. So agrees w/ usual definition.

Once you show this, can prove that derivation $X$ on $C^\infty(M)$ is determined by

derivations on each $\text{V}_x$, $x \in M$. Thus get $\text{Der}(C^\infty(M)) \equiv \text{sections } M \to T^*M$
in usual sense.

Point of Ch 7: $G$ a lie group. Then $\text{Der}(C^\infty(M)) \to \text{Der}(C^\infty(M)) G$ under action of $G$. The lie algebra $\text{Der}(C^\infty(M))$ is closed under $[E, J]$ so forms a sublie algebra.

Can determine all tangent vectors in a left invariant $V_x$. If your Nuclear Basis at $I$, so

$\text{Der}(C^\infty(M)) G \equiv \text{V}_x \oplus \text{V}_x$. Then do a computation to see that

the bracket can be thought of using the usual commutator of matrices, $f G G$. 

Point of Ch 8: Given a left inv $V_x$, find an $E$ whose derivative is that. Use near $I$ is an ODE, so it is locally solvable. By translating by gp elt, can make it a 1-para.

family globally. This is the exp map! Check: agrees w/ matrix defn (easy).

and $E, J$ comes from derivative of Ad action via exp, as before.