1. Prove: L simple \(\Rightarrow\) the only 1D repn is trivial.

2. Def.: We say that \(L\) is a central extension of \(K\) if there is a SES
\[0 \rightarrow I \rightarrow L \rightarrow K \rightarrow 0\] where \(I \subset Z(L)\).

Prove that \(L\) is nilpotent \(\iff\) \(L\) is a trivial central extension of abelian lie algebras.

3. Find the radical of
\[P_{m,n} = \begin{pmatrix} \ast & \ast & \cdots & \ast \\ \ast & \ast & \cdots & \ast \\ \vdots & \vdots & \ddots & \vdots \\ \ast & \ast & \cdots & \ast \end{pmatrix} \subseteq gl_{m \times n}.\]
What is \([L,L]\)?
Is \(L\) solvable?
Is \(L\) nilpotent?

4. Find the radical of
\[L = \begin{pmatrix} \ast & \ast & \cdots & \ast \\ \ast & \ast & \cdots & \ast \\ \vdots & \vdots & \ddots & \vdots \\ \ast & \ast & \cdots & \ast \end{pmatrix} \subseteq gl_{n \times n}.\]
What is \([L,L]\)?
Is \(L\) solvable?
Is \(L\) nilpotent?

5. Find the radical of
Heis = \[\begin{pmatrix} \ast & \ast & \cdots & \ast \\ \ast & \ast & \cdots & \ast \\ \vdots & \vdots & \ddots & \vdots \\ \ast & \ast & \cdots & \ast \end{pmatrix} \subseteq gl_{n \times n}.\]
What is \([L,L]\)?
Is \(L\) solvable?
Is \(L\) nilpotent?

6. a) Let \(W, V\) be fl-dimensional and \(\Phi: \wedge^2 V \rightarrow W\) a linear map. Prove that
\[L = V \oplus W \quad [v_1,v_2] = \Phi(v_1 \wedge v_2) \quad [v,w] = 0 \quad [v_1,v_2] = 0\]
defines a lie algebra?

b) Solvable? Nilpotent? What is \([L,L]\)?

c) Show that \(L \cong L_\Phi \iff \exists g \in GL(V) \text{ lie } GL(W) \text{ s.t. } \Phi = (W)\Phi(W)\)

d) Compute \(\dim \text{ Hom}_{GL}(\wedge^2 V, W), \dim GL(V), \dim GL(W)\). Deduce that there are
LOTS of non-isomorphic lie algebras of this form when \(\dim V, \dim W\) are large.

Humphreys Ch 2 \# 5, 6, 7  Ch 3 \# 2, Why is this different from exercise 2 above?

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