1. a) Consider $SL_2(C)$ and identify $Lie(S)$ with $R^3$. Which representation of $Lie(S)$ integrates to $S$?

b) Which reps of $C = Lie(C^*)$ integrate to $C^*$?

c) Which reps of $su(2)$ integrate to $SL(2,R)$? To $SO(3)$?

d) Which reps of $C^n$, the abelian Lie algebra, integrate to $C^n$?

e) Which reps of $sl(2;R)$ integrate to $SL(2;R)$. What is $T(sl(2;R))$?

2. If $a \in A$ (ass. alg.) is nilpotent, then $ad_a \in End(A)$ is nilpotent.

3. Let $x \in g$ (lie alg.). Let $V_cay$ be the span of the eigenvectors for $ad_x$. Prove that $V$ is a sub-lie alg. How does $V$ act on the span of the $x$-eigenvectors in a rep. of $g$?

4. Let $P$ be the vector-space w/ basis given by the power set of $\{1,2,...,n\}$.

That is, $\dim P=2^n$ w/ basis $\{v_S\}$ for $SC\{1,2,...,n\}$.

Define $e_i^* : v_S \rightarrow \sum_{i \in S} v_S$ and $f_i^* : v_S \rightarrow \sum_{j \in S} v_{S\{i\}}$.

a) Prove that this produces a rep. of $S^n$. What is $h$?

b) What is its character? How does it decompose into irreducibles (not easy!)

c) Find a combinatorial way involving something like Pascal's triangle.

d) Prove that $P \cong V_1^{\otimes n}$ as $S^n$ reps.

e) When $n=4$ find all the primitive vectors.

5. Prove (carefully) that

a) $\Delta(\lambda)$ is irreducible when $\lambda \not\in \mathbb{Z}_{\geq 0}$.

b) $\Delta(\lambda)$ is indecomposable for $\lambda \in \mathbb{Z}_{\geq 0}$.

c) Find a nontrivial extension of $\Delta(\lambda)$ and $\Delta(-\lambda-2)$ for $\lambda \in \mathbb{Z}_{\geq 0}$.

(If you just do $\lambda=0$ and $\lambda=1$ that is OK.)
6. We have drawn pictorial depiction of \( V \) w.r.t. the basis \( \{ x^a y^b : a + b = 1 \} \).
Now draw the picture w.r.t. the basis \( \{ x^a y^b \} \) where \( x^a = \frac{x^a}{a!} \).

7. Place a lie alg. structure on \( g \) s.t.: \( Xg_y, Yh \Rightarrow [X,y] \in g \).
(Tie to define this explicitly.) If \( g=\text{Lie}G, h=\text{Lie}H \), what is \( g \cap h \)?

8. Let \( U_3 = \left( \begin{smallmatrix} 1 & x \\ 0 & 1 \end{smallmatrix} \right) \in \text{SL}_3 \) and let \( \eta_3 \) be \( \text{Lie}U_3 \). Use the "matrix entry" basis of \( \eta_3 \), compute its action on \( \mathbb{C} \text{SL}_3 \) and on \( \wedge^k \mathbb{C}^3 \) for \( 0 \leq k \leq 3 \).