

Prerequisites and references

1. a) Symmetric group: generators and relations, length function, longest element, Bruhat order. Matsumoto's theorem.

b) Braid group (type A): generators, braid relations.

References:

A. Bjorner, F. Brenti. Combinatorics of Coxeter groups. Chapters 1 and 2.

B. Elias, S. Makisumi, U. Thiel, G. Williamson. Introduction to Soergel Bimodules. Chapter 1.

2. Linear algebra: exterior algebra, minors, the relationship between rank and minors of a matrix.

References:

V. Guillemin. Notes on differential forms, chapter 1. <https://math.mit.edu/classes/18.952/spring2013/docs/book.pdf>

V. Prasolov. Problems and theorems in linear algebra.

3. a) Grassmannians and flag varieties (type A): definition, Plücker coordinates, coordinate charts, Schubert cells (including explicit parametrization). Plücker relations.

b) Flag varieties as cosets G/B . Bruhat decomposition.

References:

E. Miller, B. Sturmfels. Combinatorial commutative algebra, section 14.

M. Gillespie. Variations on the theme of Schubert calculus. <https://arxiv.org/abs/1804.08164>

S. Billey. "Basic Schubert calculus" slides: <https://sites.math.washington.edu/~billey/talks/SchubertCalculus.pdf> and "Lectures on Schubert varieties" <https://sites.math.washington.edu/~billey/classes/schubert.notes/notes.pdf>

W. Fulton. Young Tableaux. With Applications to Representation Theory and Geometry, part III.

Optional reading and more references:

1. Cluster and braids online seminar notes (Summer-Fall 2021): <https://sites.google.com/view/clusters-braids-seminar/#h.8pehghic1fj>

2. A. Bjorner, F. Brenti. Combinatorics of Coxeter groups. Chapter 5: Kazhdan-Lusztig and R -polynomials.

3. L. Escobar. Brick manifolds and toric varieties of brick polytopes. arXiv:1404.4671

4. S. Fomin and A. Zelevinsky. Total positivity: tests and parametrizations. arXiv:math/9912128

4. S. Fomin, L. Williams, A. Zelevinsky. Introduction to Cluster Algebras. Chapters 1-3 arXiv:1608.05735 and Chapter 6 arXiv:2008.09189

5. M. Gorsky, J. Simental. Clusters and Weaves. Notices of the American Mathematical Society. <https://www.ams.org/journals/notices/202408/rnoti-p1004.pdf>

6. A. Knutson, T. Lam, D. Speyer. Positroid varieties I: juggling and geometry. arXiv:0903.3694

7. A. Postnikov. Total positivity, Grassmannians, and networks. arXiv:math/0609764