EXERCISES FOR LECTURE 11

1. Main exercise

Exercise 1. In this exercise we take $G = \mathrm{PGL}_2$, with the same choices of Borel subgroup and maximal torus as in Lecture 9.

(1) Describe explicitly the intersections $\mathrm{Gr}^\lambda \cap S_\mu^+$ and $\mathrm{Gr}^\lambda \cap S_\mu^-$ for $\lambda \in \mathbf{X}_+^\vee$ and $\mu \in \mathbf{X}^\vee$. (*Hint*: You have essentially already done this computation in the exercise for Lecture 2.)

(2) Check the formulas

$$\dim(\mathrm{Gr}^\lambda \cap S_\mu^+) = \langle \rho, \lambda + \mu \rangle \quad \text{and} \quad \dim(\mathrm{Gr}^\lambda \cap S_\mu^-) = \langle \rho, \lambda - \mu \rangle$$

in this case.

(3) Check in this case that we have

$$\mathsf{H}_c^n(S_\lambda^+, (s_\lambda^+)^* \mathcal{F}) = 0 \quad \text{and} \quad \mathsf{H}^n(S_\lambda^-, (s_\lambda^-)^! \mathcal{F}) = 0$$

unless $n = \langle \lambda, 2\rho \rangle$, for any $\mathcal{F} \in \mathrm{Perv}_{G(\mathscr{O})}(\mathrm{Gr})$. (*Hint*: Use the properties proved in the exercise for Lecture 9.)

(4) Check in this case that for any $\mathcal{F} \in \mathrm{Perv}_{G(\mathscr{O})}(\mathrm{Gr})$ and any $n \in \mathbb{Z}$ we have

$$\dim \mathsf{H}^n(\mathrm{Gr}_G, \mathcal{F}) = \sum_{\substack{\lambda \in \mathbf{X}^\vee \\ n = \langle 2\rho, \lambda \rangle}} \dim \mathsf{F}_\lambda(\mathcal{F}).$$

2. Additional exercise

Exercise 2. Try to complete the proof of the geometric Satake equivalence in the case of PGL_2 (either by completing the sketch of the general proof, or by inventing a more direct specific argument in this case).