

# Iwahori-Whittaker & Arkhivov-Bezrukavnikov (1)

## I. Whittaker sheaves

Work over  $\overline{\mathbb{F}_p}$ , w/  $\mathbb{C}$ .

$A'$  is NOT simply-con:.

$$x^D - x : A' \rightarrow A'$$

- is - surj.  
 - proper (in fact, finite)  
 - "local" homeo. because derivative is always  $\neq 0$ .

This is a Galois cov., Gal gp =  $\mathbb{Z}/p$ .

$\rightarrow$  Get a loc sys  $\underline{AS}$  (of "1" vec sp.)  
 on which Gal gp acts by  $p^k$  roots of 1.

Look at  $U = \begin{bmatrix} 1 & * \\ & 1 \end{bmatrix} \subset \mathbb{P}^1$ .

Orbits:

A sheaf  $\mathcal{F}$  on  $\mathbb{P}^1$  is:

- equivariant if:  $U \times \mathbb{P}^1 \xrightarrow{a} \mathbb{P}^1$   
 $\downarrow \text{pr}$

Recall  $a^* \mathcal{F} = \text{pr}^* \mathcal{F}$ , etc. 2 IC's, other obj.

- Whitt.-equivariant if  $a^* \mathcal{F} = \underline{AS} \otimes \mathcal{F}$ .
- $\uparrow$  NOT possible if  $\mathcal{F}$  supported on 0-divisor orbit.
- $j_! \underline{AS} = j_* \underline{AS}$ .
- $D^b(\text{Whitt. eq. on } \mathbb{P}^1) \simeq D^b \text{Vect}_{\mathbb{C}}$ .

~~Fact~~ Exercise. "Whittaker-averaging on  $\mathbb{P}^1$ ".

## II. Iwahori-Whittaker sheaves

Work on  $Fl_G = G(\mathbb{R})/I$ .

Also:  $I^- =$  opposite Iwahori.

"  
 $T \times I_u^-$   
 torus  $\leftarrow$  co-divisor unipotent.

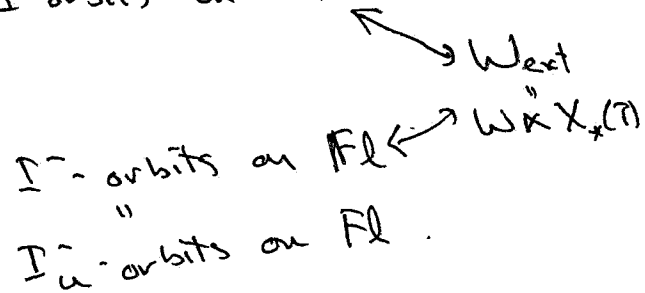
Take "generic"  $\chi: I_u^- \rightarrow A'$   
 homom.

Look ~~at~~ Goal: Say something about twisted  $I_u^-$ -equiv.

w/it.  $\chi^* \underline{AS}$ .

Generalities:

$I$ -orbits on  $Fl$



Lemma. ~~Let~~ Let  $w \in Wext$ . TFAE:

- 1)  $I_u^- \backslash I/I$  admits a  $\chi^* \underline{AS}$ -tw. equiv. loc. sys.
- 2)  $I_u^- \backslash I/I$  is dense in  $G(\mathbb{O}) \backslash I/I$  ( $\simeq$  aff. bundle over  $G/B$ )
- 3)  $w$  is minimal in the coset  $Ww$ .

Notation:

$$\text{Perv}_{Iw}(Fl) = \text{Perv}_{I_u^-, \chi^* \underline{AS}}(Fl)$$

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Irr. obj in  $\text{Perv}_{IW}(Fl)$   
 $\leftrightarrow W/W_{ext} \leftrightarrow X_*(T)$

$$K(\text{Perv}_{IW}(Fl)) = \mathbb{Z}[W] = K(\text{Coh}^{\check{G}}(\tilde{N}))$$

Thm [Arkhipov - Bezrukavnikov]  
 $\exists$  equiv. of tri. cat

$$D^b \text{Perv}_{IW}(Fl) = D^b \text{Coh}^{\check{G}}(\tilde{N})$$

$$D^b_{IW}(Fl) \quad D^b \text{Coh}^{\check{B}}(\tilde{u})$$

Plan ~~First~~ Look at "easiest" obj on coherent side  
 - free sheaves  $V \otimes \mathcal{O}(\tilde{u})$   
 $\uparrow$  or  $\mathcal{O}(\tilde{N})$   
 - (dominant) line bdl's  $\mathcal{P}_\lambda \otimes \mathcal{O}(\tilde{u})$   
 or  $\mathcal{O}(\tilde{N})$

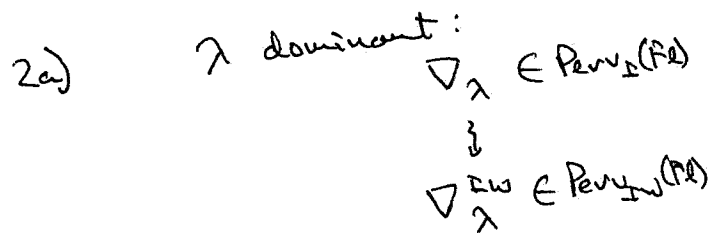
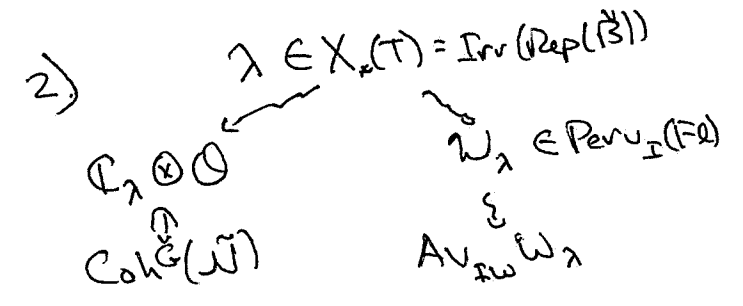
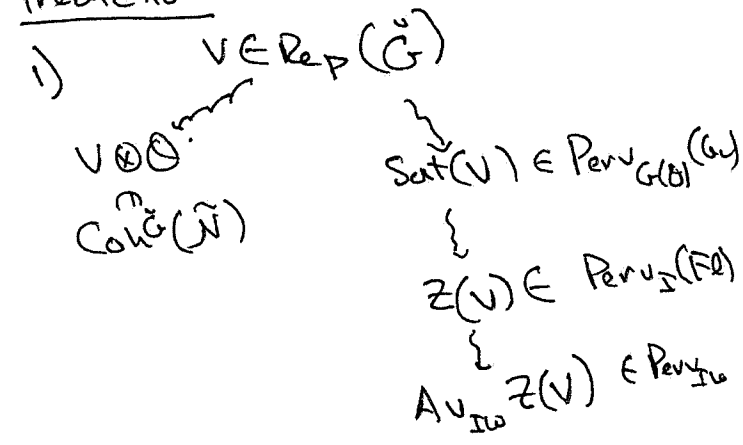
- Predict where they should go on perverse side
- Use prediction to define a functor. This is the hardest part.

Once the functor is defined, checking that it's an equiv. is not bad.

III. Carrying out the plan

Key tool: "Whittaker averaging"  
 $A_{V, IW}: D^b_I(Fl) \rightarrow D^b_{IW}(Fl)$   
 $\uparrow$  p'-version in exercise.

Predictions



IV. Why Iwahori-Whittaker?

