Lecture 13: main exercises

Exercise 13.1. Let $Q = 1 \to 2 \to 3$.

(a) Compute the exchange graph of A_Q, drawing it as the edges of a polytope in 3-space. Use the correspondence with triangulations, which in this case says the vertices of the graph correspond to triangulations of a hexagon.

(b) Compute the cluster variables in A_Q. How many are there?

(c) Show that mutating at 1, then 2, then 3 results in a quiver which is the same as the original one, up to permuting the labels of vertices. Such a periodic sequence of mutations gives rise to an automorphism of the exchange graph, and of the cluster algebra (i.e. by sending the initial cluster variables to those of the cluster obtained via the mutation sequence). What is the order of this automorphism in this case?

Lecture 13: additional exercises

Exercise 13.2. Let S be a surface with boundary and a collection of marked points on the boundary. A triangulation will mean a triangulation where the vertices of the triangles are on the marked points (so e.g. "a triangulation of a pentagon" would also be described as "a triangulation of a disk with 5 marked points on its boundary"). A triangulation determines a quiver whose vertices are the internal edges of the triangulation, and whose edges are drawn inside the triangles as in class (so e.g. there will be at most 3 edges in a triangle, and this happens exactly when that triangle has no edges along the boundary).

(a) Let S be an annulus with one marked point on each boundary component. Determine the associated exchange graph. How many different quivers appear?

(b) Let S be an annulus with one marked point on one boundary component and two on the other. Determine the associated exchange graph. How many different quivers appear?